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TEACHING STRATEGIES TO ACHIEVE COMPETENCIES

INTRODUCTION :

- How is the teaching mathematics in our schools?
- Why are lot of children afraid of mathematics?
- Though the students are getting good marks, how many of them have standards of mathematics?
- The teachers have been given many hand books and trainings for the last 10 years. To what extent these handbooks and trainings brought changes in the process of teaching learning process?
- What should be done to bring changes in the teaching methods?
- What should we do to make the students to enjoy mathematics learning?
- How a teacher should prepare to carry out exciting and successful learning activities in the classroom?

Let us think about the above questions. If these questions incite us to think and make us to learn, we can carry out interesting activities which make learning joyful. Thus mathematics learning will be simplified. Expected competencies and skills are developed as we expected. For achievement of academic standards in the students, let us discuss the strategies should be adopted in the class room.

According to educational psychology, learning means "the permanent relative change in behaviour" that means a child has already innate abilities in him which will be developed as skills in

course of learning. But in the present class room, the teacher shares some information with the children and memorising this information is treated as knowledge. After memorising, whosoever presents this knowledge in oral or written aspects is considered to be intelligent and talented. The main reason for this is nothing but present teaching method.

In accordance with the instructions and suggestions that laid in NCF-2005, APSCF-2011 and RTE-2009, the subject wise skills should be developed in the students along with the basic understanding of the concepts in the subjects. In case of mathematics, teaching learning activities should develop problem solving, reasoning-proof, communication, connections and representation-visualisation skills in the students, according to mathematics position paper. The skills developed in the classroom should be useful in their day-to-day lives and can be associated with other branches of knowledge. The application of these mathematical skills makes them able to invent new things.

Let's discuss the competence wise strategies to be adopted which develop the mathematics skills in the students.

PROBLEM SOLVING :

Problem solving in mathematics means whenever a problem arises it should be solved by a process or method. Problem solving strategies has to be habituated by the students to develop problem solving competency in them. If it is habituated, the children can solve any problem on the given concept.

Steps in problem solving:

- Reading the problem
- Identifying the components in the data of the problem
- Identifying "what is to be found?"
- Separating the required data
- Understanding the concept involved in the problem
- Visualising as many as strategies to solve the problem
- Selecting a strategy among them (selecting a formula and procedure to solve)
- Implementing the strategy and computing by substituting the values in the data
- Arriving to the solution
- Verifying the solution
- Testing another ways of solution

But, to make the children understand the problem solving method

- Teacher should write the problem on the blackboard
- Ask the children to read the problem on the blackboard
- Discussing with the students about the data given in the problem and what to be found?
- Identifying the concepts involved in the problem and relation between them by the children
- Writing the required data on the black board by identifying them by the children
- Deciding the strategies to solve the problem and writing the formulae on the black board on discussing with the children
- Substituting the values in the appropriate formula and solving the problem with various computations through discussing with the children
- Arriving to the solution
- Verifying the solution or result through the discussion with the children
- Giving opportunity to the students to ask doubts in the process of problem solving and clarifying them through discussion.
- Like this, students should be habituated the problem solving strategies gradually through discussions with them.

Then, another model problem should be given to the students for assessing the comprehension of the students in problem solving strategy. For that we have to

- Write a model problem on the board (similar but not same)
- Ask the children to write the problem in their note books and solve it individually.
- Observe procedure of problem solving by the students personally by moving around in the classroom
- After problem solving, ask the student to discuss in the groups about their problem solving.
- Observe their discussion of problem solving by the students by moving around in the classroom identify their misconceptions and errors made by them
- Small errors made by the children in the problem solving can be clarified in the groups

- Misconceptions can be written on the board and can be understood by applying inductive or deductive logics. And these logics should be discussed in the whole class and thus clarify the misconceptions in problem solving.
- If most of the children in the class room are committing more mistakes, the teacher should write another similar problem on the board and discuss strategies of solving problem.

The children must be able to solve the problems individually on their own. Therefore the students should understand strategies of problem solving and they should be able to explain the process of solving any problem. The teacher should discuss each step of the problem and connections in between each step and antecedent step. These steps should be elicited through the discussions so that they can explain the process of problem solving. The students should be given chance to solve the problems in the exercises on their own. Thus, they should be encouraged to solve any problem on their own.

REASONING PROOF :

Instead of problem mechanically, in the class room children has to explain appropriate reasons through logic in the understanding of concepts and also in problem solving.

For this, the teacher in the class room has to develop the competency of reasoning and proof among the children

The indicators for this are

- Generalizing concepts into mathematics statements or proving theorems in mathematics should be done the basis of inductive or deductive reasoning's.
- Making the hypotheses in generalizations through understanding the patterns.
- To test hypothesis in some particular cases and making conclusions.
- Giving reasons for various steps in problem solving
- Understanding the procedural pattern in problem solving.
- Verification of solution in problem solving.

All the above indicators indicate the development of reasoning and proof competency in the students. To enhance this competency among the children in the mathematics class the teacher may follow these following suggestions.

- Make the children to read comprehensively the text given in the text book to understand the concept.
- Let the students to understand the logic in the concept by discussing with peer groups

- Provide some activities to understand the concept which can be discussed in groups or conducted individually
- The understanding of the children should be discussed in the whole class.
- By asking a sequence of questions the children develop the logic in the concept.
- The children should be asked to test the hypothesis made in the concept in particular cases.
- The discussion of problem solving procedures with giving reasons to each step. This has to be done in the whole class discussion with the student to understanding problem solving strategies
- While proving the theorems the students should be habituated to give reasons for each step.

COMMUNICATION :

Mathematics has its own language. It has symbols signs and terms with good grammar. Therefore communication in mathematics is very important. In mathematics any problem solved or concept understood is based on the communication in mathematics only. Through these communications only mathematics logics can be understood. Therefore communications skills in the learning of mathematics should be developed among the students.

The indicators for communication performance are

- Reading, writing and expressing mathematical concepts or generalizations.
- Reading writing and expressing mathematical sentences through signs and symbols.
- Explaining thoughts and logics in mathematics in own sentences.
- Explaining procedures of problem solving
- Generating or preparing new problems.

The teacher should adopt these following strategies in the class room while teaching learning process to develop above mathematical communication skills

- Encourage the children to read the text given in the textbook for understanding the concept the important mathematical terms and symbols, which comes across at the time of reading should be written on the blackboard.

- For extensive understanding of the concept the teacher should provide activity to be performed in groups or individually. In this process whatever the new terms and symbols come across should be practiced by children.
- While understanding the strategies of problem solving on a concept, the teacher should be make the children read problem comprehensively and identify the terms and symbols involved in it.
- After problem solving the children should be encouraged to explain the procedure of problem solving and expressing different methods to solve that problem.
- After solving the problem by the children individually all through the chapter the children should be encouraged to prepare or generate new problems.

CONNECTIONS :

Connections of various concepts will be required while understanding of mathematical concepts. Whenever a problem is solved connections between the various concepts are utilized. These connections may be in between to various mathematical concepts or mathematics concepts with concepts of other subjects or mathematics concepts with concepts in daily life. Therefore in the learning of mathematics connection skill is very important to be developed among the students.

The indicators for the performances of skill of connections are

- Connecting the concepts in an area with the concepts of other area in mathematics.
- Connecting the concepts in mathematics with the concepts in mathematics with the concepts in other subjects.
- Connecting the concepts in mathematics with mathematics with the concepts in daily life.
- Connecting various concepts while problem solving.

A teacher can adopt following strategies to develop above skills in the students.

- The students read the text in the process of understanding a concept. He connects his previous notions to understand the given text in the textbook.
- Then, the student is provided an activity to understand a new concept. In this process, the student uses inductive or deductive logic to understand the concept. He needs to connect the previous notions and present experience in the activity in process of using inductive or deductive logics.
- The teacher has to use connections more comprehensively in whole class activity to have a complete understanding on the concept.

- Making use of logics which are developed in other areas of mathematics to understand a new concept. I.e. connecting a concept in an area with the concept of another area. For example, using similar triangles in understanding trigonometric ratios. Connecting or using the concepts to understand a new concept in mathematics, connecting the daily life concepts or notions to understand a concept can also be used in understanding a concept.
- The students should be habituated to see logical connections in between the things.
- Besides connections in understanding a concept, a student should also be able to connect the concepts in the process of problem solving.
- A teacher has to make the children use connections in between the concepts in solving problems.
- In the first step, the teacher writes a problem on the board and asks the students to read it. Then, he asks them to identify given information in the problem. Based on the given information, he then asks the students to identify the concepts involved in the problem.
- The teacher should elicit connections in between the concepts involved in the problem through the discussion with the students.
- Then based on these connections, the teacher habituates the students to visualise the strategies to solve the problem.

Thus, the students develop connecting skill in mathematics in process of understanding a concept or problem solving.

REPRESENTATION -VISUALISATION :

The students visualise while understanding a concept or strategy to solve a problem by connecting the things involved in it holistically. If the students are not able to visualise the things with appropriate logic, they cannot solve problems or cannot understand the concepts.

When we try to represent the visualised concept in pictorial form, we need to have representation skills. These representations make anyone to understand that concept easily. These representations may be in the form of a histogram or a curve or a straight line in graph or may be in the form of a pie chart, flow chart, constructions. These representations normally facilitate us to show complex information or more information in simpler form in the form a picture. It may look simple but it provides us much information in a simple picture. The children should show following indicators to present the skill of representation.

- Reading the information in a table, number line, pictorial diagrams, graphs, 2-D and 3-D pictures and flow charts.
- Representing the information in the form of a table, number line, pictorial diagrams, graphs, 2-D and 3-D pictures and flow charts.

A teacher may adopt following strategies to develop the representation and visualisation skill in the students.

- We should make the students to visualise the contexts or content in the textbook in the process of understanding a concept.
- The students should be encouraged to visualise the strategies of problem solving with appropriate logical connections in between the steps.
- Habituate the children to represent those visualisations in the form pictorial flow charts.
- Besides above skills of representations, there some more standard representational styles created in mathematics like graphs, pie charts, tables, etc.
- Whenever we provide learning strategies to understand standards representations, the child should be habituated follow fundamentals involved in it.
- When we represent graphs, we need decide the scale of the graph. In the case of frequency distribution table deciding class intervals should be decided.
- When a student needs to construct geometrical figures, he needs to visualise the figure in the form of a rough figure and decide the procedure to construct.
- Above discussed fundamentals should be discussed in peer groups of students and made them to understand in whole class.
- The teacher should give suggestions or directions in these processes wherever they need help and thus skill of representation should be developed in the students.

Thus students develop representation and visualisation skills and solve the problems that come across in his life. They also present their skills wherever the necessity arises.

Mathematical competencies develop confidence in the child to perform any activity in his daily life actively. He always uses logics to perform better. Mathematical competencies are useful for a child to save the time in various activities. Hence, a teacher can mathematise the life of a child by developing above competencies in him.



CHAPTERS IN TENTH CLASS SYLLABUS -KEY ELEMENTS

Mathematics has been developed as per the demand of requirements of mankind. The invention of numbers and their operations have been defined as the requirements of human beings demanded. When the demands of the society increased and these numbers and operations became insufficient, the branches of the subject like algebra, trigonometry, data handling, probability etc. have evolved out the needs of changing society. The concepts like solutions of equations, logarithms, trigonometric ratios, sets, etc. made problem solving in various situations easy. But, is mathematics problem solving only? Of course not! It has its own beauty in it in the form of logic everywhere. This beauty has been expressed in various contexts by many wits (like Euclid, Pythagoras, Arybhata, Bhaskaracharya) in the form of axioms, theorem, statements, formulas, etc. These are stated either by applying inductive logic or deductive logic by connecting various earlier various general statements and testing them in some peculiar cases. As it is mentioned earlier, logic is like spine in all these processes. Whatever it is stated in mathematics, it is not stated without logic. This logic makes us enjoy mathematics.

As we discussed earlier, mathematics should be useful for us to solve problems which come across in our daily life and have fun. Mathematics should also encourage the children to test the logical statements and enjoy with the results. But, how is the learning of mathematics in our classrooms? Are your children in classroom enjoying the beauty in mathematics? Is a student having fun with mathematics?

If we want to achieve the objectives in the children which are expected in mathematics learning, is it enough to teach the problems from only textbook? The answer may be no. We cannot achieve the

academic standards decided for a class, if we teach the only problems from the mathematics textbook. We, the teachers should understand the backdrop of each concept mentioned in the textbook to enjoy the teaching learning process in the classroom.

Therefore, we should know about the key elements in each chapter to take up successful strategies in the class room. Hence, let us discuss about the key elements in the chapters in the mathematics textbook of tenth class.

1. REAL NUMBERS

When we look at the first chapter Real numbers, the properties of rational and irrational numbers have been discussed in the beginning. Some basic theorems or general statements were stated on the basis of inductive or deductive logics by using some earlier basic properties of rational and irrational numbers. If they comprehend these logical statements, they could explore some more logics and evolve as good mathematicians.

Fundamental theorem of arithmetic is discussed initially in the chapter. It is not only discussed with respect to solve problems but also it is discussed in the view of stating different types of statements. Here, we as mathematics teachers should only encourage the students to generalize and state mathematical statements on the basis of their observations. We should also discuss various types of statements like converse, inverse, etc. Similarly, we should encourage students to similar kinds of statements on the basis of properties of rational and irrational numbers.

Moreover, logarithms were discussed in the later part of the chapter. Thus we have to understand the backdrop and key elements of the concept. Is it necessary to learn logarithms at tenth class stage? What happens, if they won't learn logarithms? If we could answer these questions, we would easily understand the backdrop and the key elements of logarithms in the tenth class textbook.

Let's understand it by an example

How would children respond, if they are asked these questions?

$$10^4 = 10000$$

$$10^2 = 100$$

$$10^1 = 10$$

$$10^{1/2} = ?$$

Moreover, what should be the value of x , if $10^x = 2$?

What should be the value of x , if $10^y = 3$?

Are there any ways to find the values of x or y ? What can be the value of $x+y$?

When we allow the children to think over these types of questions, they may understand the necessity of logarithms easily. Logarithms would be merely a mechanical procedure, if we introduce them merely as a process of converting a power into a logarithmic form.

Questions for teachers:

- Is there any relationship in between progressions(A.P &G.P.) and logarithms?
- What are the limitations of values of base and logarithmic number? How can u say?

2. SETS

Fundamentals of sets and terms & symbols related to set theory are discussed in Sets chapter besides operations on sets. Actually, these concepts of sets are base for abstract algebra. Thats why, the children can understand how Cantor introduced sets in higher education only. But, as recent globalization (higher studies like commerce, arts, management) demanded for the basics of sets, we are forced to introduce the basic concepts of sets. Therefore, we confine ourselves to the basic concepts of sets at this level of learning.

3. POLYNOMIALS

Children of tenth class have notions of polynomials already. Those knowledge of polynomials utilized here for extended learning. Zero values of different types of polynomials in single variable have been discussed in the beginning of the chapter. But, a basic question is, what is the need of finding zero values for polynomials? Are they useful in our daily life situations or any other areas like business, management, technology, etc. When we could answer these questions, we would try to find them in many ways. A few of the simple ways of finding zero value for a polynomials are drawing graphs, relation between their coefficients, etc.

Lets discuss the relation between the coefficients of the polynomials and their zeros

Let $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n$ is polynomial of nth degree in single variable.

Here, $a_0, a_1, a_2, \dots, a_n$ are coefficients. If $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ are zero values.

Then, we can generalise like this.

$$\sum \alpha_1 = \frac{-a_1}{a_0}$$

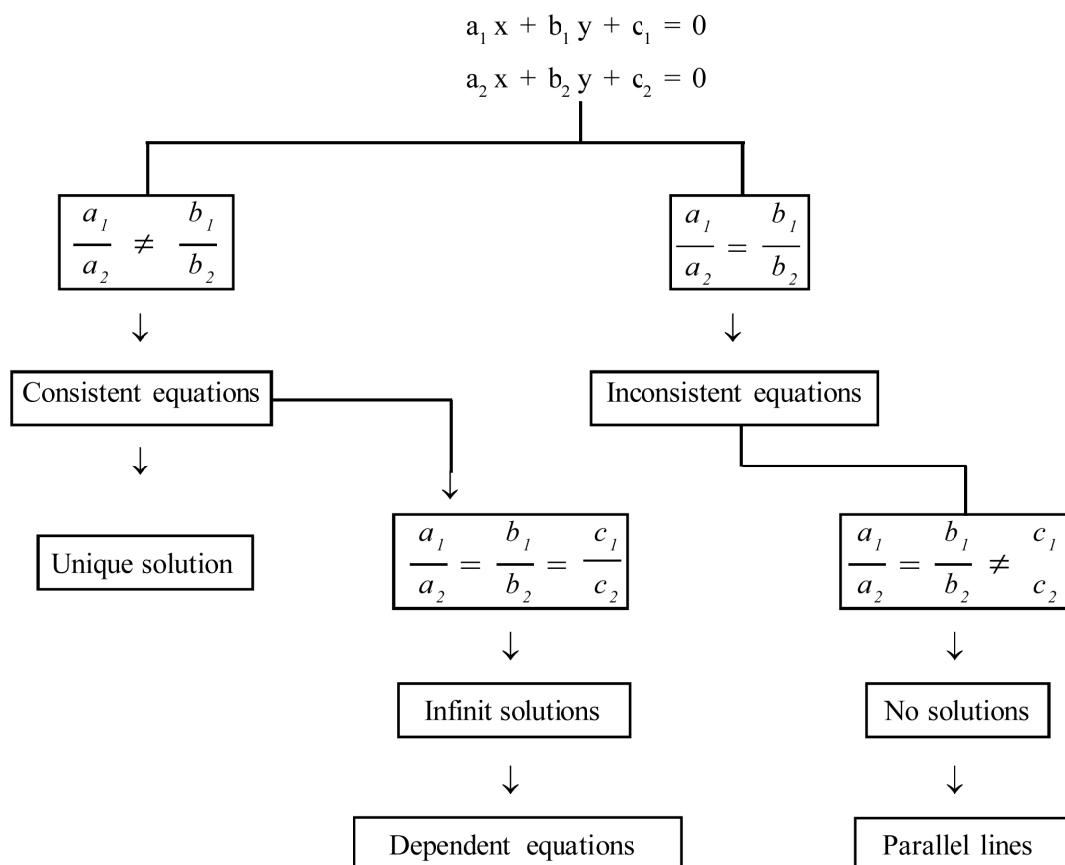
$$\sum \alpha_1 \alpha_2 = \frac{a_2}{a_0}$$

$$\sum \alpha_1 \alpha_2 \alpha_3 = \frac{-a_3}{a_0}$$

$$\sum \alpha_1 \alpha_2 \dots \alpha_n = (-1)^n \frac{a_n}{a_0}$$

4. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES:

Solutions of different equations shown us simple ways of problem solving in various situations of our daily life. The value of an equation doesn't change when a value is added or subtracted both sides. And also the value is not changed when it is multiplied or divided by a non-negative number. This property of equations is very much useful in the elimination process of variables. The skill of eliminating a variable determines the skill of solving equations most of the times. But, all these processes have been discussed in previous classes. Here, the objective of this chapter is to know the types of pairs of linear equations on the basis of their solutions. And it can be related with the nature of coefficients in the following way.



5. QUADRATIC EQUATIONS:

Quadratic equations are considered as extension of polynomials in this textbook. Can it be said that if a child has the ability to solve a quadratic equation, he has competencies expected in him? The logic of solving a quadratic equation provides us many ways of solving many complex problems. Chapter begins with the communication in mathematics in our daily life. The beginning of the chapter discusses conversion of daily life situational problems into the quadratic equations. This is considered as a very important skill in mathematics. Then, it is continued discussion of various types of solving procedures of quadratic equations. We, the role of teachers is to make children to find the connections in between them. Besides this, children are encouraged to generalize the relation between nature of roots and coefficients. This gives them joy while deciding nature of roots without finding them. Here, the students should be allowed to think about the importance of the "discriminant" and why it should be considered. We have to make them visualize the nature of roots and their uses.

6. PROGRESSIONS:

The concepts of progressions give us logic of solving problems related to patterns as well as joy of learning. Concept of progressions can be used for the building concept of numbers and algebra. When we look at the nature mathematically, many things surprise us with their patterns. Sunflower, —, reproduction of rabbits, etc. are utilized in the process of introducing progressions in this textbook. Then A.P. and G.P. were discussed gradually. Finding general term and sum of the terms in a progression are discussed in this chapter. Here, we must understand that the child should be habituated generalization process and formulation process in handling progressions. These skills will be useful when students handle concepts of algebra, arithmetic, geometry, etc. and formulate some formulae in the due process.

7. COORDINATE GEOMETRY:

Coordinate geometry facilitated us in solving problems of geometry with the help of concepts of algebra and vice versa. Coordinate geometry has taken birth by contributions of great mathematicians like Rene- de-carte and Format in mathematics. There are some exciting stories which are useful in our teaching learning process to increase attention of students towards mathematics. This chapter does not only discuss the formulae and solving of problems using them but it discusses the backdrops of the formulae. The distance between the given two points, section formula and its uses as midpoint, trisection points and centroid were discussed in the beginning of the chapter. The formulae are not directly introduced but they have been gradually deduced from simple to complex formula. Then after, finding the area of a triangle when three vertices of a triangle are given has been discussed. Collinear property

of collinear points has been discussed based on three properties viz. distance between points, area of the triangle and slopes three parts of the line. Finally, the slope of a line has been elaborately deduced and ways to find slope have been discussed.

8 & 9 SIMILAR TRIANGLES & TANGENTS AND SECANTS TO A CIRCLE:

Similar triangles & tangents and secants to a circle are the two chapters which are discussed in tenth class mathematics textbook I geometry area. These chapters are extension of learnt concepts in geometry in previous classes. The student understands the basics and some properties in the form of theorems in geometry in previous classes. The student learns deductive logics in geometry in the form of proving theorems in this class. He connects different previously known properties or rules in geometry to prove some theorems. Most of the discussion in these chapters is based on similar triangles & properties of tangents and secants to a circle.

Finally, finding areas of segments of a circle and areas of different beautiful designs involving segments of circles are discussed at the end of the chapter. The student develops problem solving strategies by solving problems on different combination of solids in mensuration. He also develops the skill to connect the things mathematically. This skill helps him in higher studies like various engineering courses. He not only learns the concept but also he enjoys the logic in the chapter by finding areas of different designs involving segments of circle.

10. MENSURATION:

Mensuration in tenth class is the extension of the learnt concepts in mensuration ninth class. The student develops the logics in basics of mensuration in the ninth class. The student has already learnt the concepts of volume and surface areas of different regular solids like cube, cuboid, cylinder, cone, sphere, etc. The child has to find the volumes and surface areas of combination of solids. The child has to visualize the parts of a solid into different regular solids and visualize the strategies to find the volume and surface areas of the solid. When we look at our surroundings we come across various shapes which do not contain a single shape. We need to find various measurements of these solids to solve problems in our daily life like painting, finding capacity, etc.

11&12. TRIGONOMETRY & APPLICATIONS OF TRIGONOMETRY

Many geometric concepts helps us in solving problems we come across in our daily life in various situations. We may know the properties of similar triangles and theorems regarding them formally, but they may not helpful in connecting the ratios of sides in a triangle with its angles. When we proceed to define these relationships many problems in mathematics are facilitated to solve. This procedure was in use during 500 B.C. Different astronomical calculations were done by using this procedure.

For this, the child has to visualize right angle triangles in his daily life situations. Then he should be able to identify opposite and adjacent sides of an angle. This competency helps the children in understanding the meanings of basic trigonometric ratios. They can easily visualize these ratios in terms of functions. These ratios make problem solving easy related to heights and distances in our daily life. In this chapter the child is provided to think about the trigonometric ratios of special angles like $30^\circ, 45^\circ, 60^\circ$, etc and the problems related to them.

This chapter is not only confined to define the trigonometric ratios and find their values for some particular angles but also it discusses the deducing of new formulae and identities. Generally the students are forced to memorise the definitions of trigonometric ratios, their values for particular angles, identities, etc., though they do not know the meanings of what they are memorizing. If they understand the logic hidden in the definitions and formulae they can easily explore, invent many things on their own. They can even enjoy the learning of mathematics.

Generally, we the teachers make the students solve the problems of trigonometry regarding heights and distances. But these problems have utilitarian values and they need to be applied in daily life situations. If a student can visualise what he has learnt, he enjoys solving problems regarding heights and distances.

13. PROBABILITY

In our daily life we come across many situations in which we have to estimate the results. In the areas like technology, business, surveys the results have to be estimated and we have to take decisions based up on our assumptions. Probability helps us in estimating these results. Probability has been introduced in the mathematics of class 9 informally. But basic understanding of probability has been

discussed in class 10. Moreover basic concepts like mutually exclusive, complementary events have been discussed so that it can be useful in further studies.

14. STATISTICS:

In human evolution process, many systems have been created as the necessities have been increased of the society. Market system, technology, government machinery, schools, etc. have been introduced and undergone many changes as the needs of the society. In the management of these systems much data is to be collected, manipulated, concluded and interpreted. For all this process, statistics helps us to solve many problems in these systems.

Measures of central tendency have been discussed in statistics chapter in present tenth class mathematics textbook. It not only meant to calculate on the basis of some formula, but also it is discussed with its meaning elaborately. Student has been given opportunity to interpret on result. It is not only discussed finding the median by formula, but also by drawing ogive curves. This also helps the children in visualizing the measures of central tendency. Can we find mean or mode by using these ogive curves? If yes, how? If no, why? Discuss.

Appendix: (Mathematics modelling)

We have discussed the traditional areas of mathematics and the concepts included in each chapter in each area. We also discussed the key elements and focusing competencies for each concept. Mathematics modelling is introduced first time in the history of tenth class mathematics textbooks. Modelling in mathematics is very important strategy which is useful in solving some unbelievable problems. We think some problems are difficult to be solved but proper modelling the situation can be useful in solving the problem. For example, finding the distance between the sun and the earth can be solved by modelling of the situation. Or finding the height of Mt. Everest is another example of modelling.

Some unbelievable problems are discussed in the beginning of the chapter which creates eagerness in the children to proceed further reading the chapter. Connecting the various operations, visualizing the strategies to solve a problem, selecting a procedure and representing it in the form of a flow chart are the steps in mathematics modelling. Flow chart representation of the problem solving is nothing but the mathematics modelling. This mathematics modelling skills are very much useful for a student in his further studies like engineering, management, administration etc. or respective working areas.

CONCLUSION:

We should have a good understanding about how to understand the various concepts in each chapter, how to handle activities in the classroom, how to make the children to think, how to conclude, how to encourage the children to solve the problems in exercises besides understanding the key elements in each chapter. Though the textbook gives us freedom and flexibility in deciding activities to understand concepts, “instructions to teachers” is given at the end of the chapter for designing better activities to understand concepts. We can utilize these instructions as a guide to teachers’ better maneuvering activities.

“Instructions to students” are provided at the backside of the cover page for better usage of the textbook besides instructions to teachers. The role of the teacher and students, purpose of various exercises, procedures of solving them, etc. are provided in these instructions to the students. For better learning of mathematics, the students should read instructions to the students and follow it in the classroom transactions.

We have discussed the key elements in all the chapters of tenth class mathematics textbook till now. Besides this, we have also discussed the strategies, backdrops of each concept. Let us hope that this portion of chapter may help us in better achieving of the objectives in mathematics.