

D. Golden Ratio) :

The number series which starts with $F_1=1$ & $F_2=1$ and progressively continued as $F_{n+2} = F_{n+1} + F_n$ is called Fibonacci numbers. That is $F_1=1, F_2=1, F_3=2, F_4=3, F_5=5, F_6=8$. Rational numbers $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}$ are came from $\frac{F_{n+1}}{F_n}$. Write them in ascending order and decide the place of $\frac{13}{8}$.

If we observe the above ratios (rational numbers) we can note that they are reaching a limit particularly 1.6. this is given in the chapter "Ratios" as Golden Ratio. Again we will discuss it in the chapter "Rational Numbers".

Let us extend the Fibonacci number series. In this series only 144 is the perfect number. Is there any other perfect square number in this series? The answer is not yet found.

E. Middle Numbers :

- (i) The middle number / average of the numbers " a and b" is $\frac{a+b}{2}$. Which lies between 'a and b', i.e., $a < \frac{a+b}{2} < b$. For example the middle number of 6 and 10 is $\frac{6+10}{2} = 8$ and $6 < 8 < 10$.
- (ii) Middle number of a and b means it is bigger than 'a' and smaller than "b". For example the numbers 3,4,5,6,7 are lie between 2 and 8
- (iii) If we write the numbers between 'a and b' in ascending or descending order then the middle most term is called median of 'a and b'. If $a < x < y < z < b$ then 'y' is median.
- (iv) If $\frac{a}{b}, \frac{c}{d}$ are two rational numbers and b,d are positive numbers then $\frac{a+c}{b+d}$ is called the midiant of $\frac{a}{b}$ and $\frac{c}{d}$. Further

In the above, specific principles are given for median and mediant. These are used to write rational numbers, between given any two rational numbers.

Ex: middle term of $\frac{2}{3}$ and $\frac{3}{4}$ is $\frac{\frac{2}{3} + \frac{3}{4}}{2} = \frac{\frac{8+9}{12}}{2} = \frac{17}{24}$ and
 mediant is $\frac{2+3}{3+4} = \frac{5}{7}$

if $\frac{a}{b} < \frac{c}{d}$ then $\frac{a+mc}{b+md}$ (m is any number) gives midiants . By changing the value of 'm' we will write as many as rational numbers

Prove that $\frac{a}{b} < \frac{a+mc}{b+md} < \frac{c}{d}$

F. Farey Numbers :

From '0 to 1' means the fractions from 0/1 to 1/1 is called Farey sequence. It is shown by F_n . In these fractions the denominator is always less then 'n'. This is called nth Farey sequence.

Ex: F_1 R $\frac{0}{1}, \frac{1}{1}$
 F_2 R $\frac{0}{1}, \frac{1}{2}, \frac{1}{1}$
 F_3 R $\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}$
 F_4 R $\frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, 1$

Write the fraction of F_5 and F_7

F_n is called nth Farey fraction.

G. Terminated , non-terminated, recurring, Non- recurring decimal numbers:

The fractions which are in the form of $\frac{p}{10^n}$ are called decimal fractions.

- (i) If the denominator of decimal fractions is observed it is found that 2 and 5 are factors of the denominator. So if the denominator is in the form of $2^m \cdot 5^n$, then the rational number $\frac{a}{b}$ can be written as terminating decimal number

In $\frac{a}{b}$, if $b = 2^m \cdot 5^n$ and $m \geq n$, then

$$\frac{a}{b} = \frac{a}{2^m \cdot 5^n} = \frac{a \cdot 5^{m-n}}{2^m \cdot 5^n \cdot 5^{m-n}} = \frac{a \cdot 5^{m-n}}{2^m \cdot 5^m} = \frac{a \cdot 5^{m-n}}{10^m}$$

Since $m - n \geq 0$, 5^{m-n} is a integer and $a \cdot 5^{m-n} = c$

So $\frac{a}{b} = \frac{c}{10^m}$

Therefore write 'c' and arrange decimal point according to the value of 'm'

Ex: $\frac{17}{40} = \frac{17}{2^3 \cdot 5} = \frac{17 \cdot 5^2}{2^3 \cdot 5^3} = \frac{17 \cdot 25}{10^3} = \frac{425}{10^3} = 0.425$

Also 40) 17 (0.425

$$\begin{array}{r} 0 \\ \hline 170 \\ 160 \\ \hline 100 \\ 80 \\ \hline 200 \\ 200 \\ \hline 0 \end{array}$$

Apply both the methods and write steps.

- (ii) In $\frac{a}{b}$, if b is not equal to $2^m \cdot 5^n$, then every remainder is one of $1, 2, \dots, b-2, b-1$. So the remainder repeats at one step and from there it will be continued i.e., the previous remainders will be repeated in the same order again and again. So the decimal part of the rational number whose denominator is not in the form of $2^m \cdot 5^n$ is not terminated and repeat again and again

$$\begin{array}{r}
 \text{Also } 7) \quad 10 \quad (1.428571 \\
 \quad \quad \quad 7 \\
 \hline
 \quad \rightarrow 30 \\
 \quad \quad \quad 28 \\
 \hline
 \quad \quad \quad 20 \\
 \quad \quad \quad 14 \\
 \hline
 \quad \quad \quad 60 \\
 \quad \quad \quad 56 \\
 \hline
 \quad \quad \quad 40 \\
 \quad \quad \quad 35 \\
 \hline
 \quad \quad \quad 50 \\
 \quad \quad \quad 40 \\
 \hline
 \quad \quad \quad 10 \\
 \quad \quad \quad 7 \\
 \hline
 \rightarrow \quad 30
 \end{array}$$

The remainder '3' came again i.e., from here the remainders get repeated in the same order.

H. Therefore the rational number which is in the form of $\frac{a}{b}$ can be written in the form of terminating decimal or non-terminating and recurring decimal number and the converse is also holds good.

I. Both are same: we know that

$$1 = 0.9999 \dots \quad \text{R } 0.\overline{9}$$

$$0.1 = 0.0999 \dots \quad \text{R } 0.0\overline{9}$$

$$0.01 = 0.00999 \dots \quad \text{R } 0.00\overline{9}$$

So every terminating decimal can be shown as non-terminating and recurring decimal.

We know that the limit point of a polygon if the number of sides goes on increasing is a circle. But we also know that the number of sides of a polygon is increased for any number it would not be a circle. So generally we may get a doubt that even though the number of 9s is very large in 0.9999..... How is it equal to 1?

If $0.\overline{9}$ and 1 are two different rational numbers there exist other rational numbers between them (Why?), one of them may be their average.

The average of $1, 0.\overline{9} = \frac{1 + 0.9999 \dots}{2} = \frac{1.999 \dots}{2} = 0.999 \dots = 0.9\overline{9}$ i.e., we got $0.\overline{9}$ again.

Therefore there is no other rational number between 1 and $0.\overline{9}$

$$\therefore 0.\overline{9} = 1$$

a. Irrational Numbers :

Pythagoras and his followers established a secret institution to find new investigations. They found many new things and kept secret with themselves. Their concept was all numbers are rational

numbers. But the length of the hypotenuse when it was compared with other sides became as irrational number in isosceles right angle triangle. Pythagoreans reacted very seriously on this and kept it secret. If anyone told about it he would be arrested, punished and jailed in a ship. Proclus, who had written a comment on "The Elements", said that: some of the persons among who opened their mouth about the secret of irrational numbers were died in a ship and remaining people were kept in an Island to be beaten by the ways of the ocean.

b. Real Numbers :

All numbers which are used to show the points on the number line are real number i.e., all the numbers on the number line are real numbers

c. Let us recall the definition of the rational number. We called that, The numbers which can be written in the form of $\frac{a}{b}$ (a and b are integers and $b \neq 0$) or the decimal numbers which are non-terminating and recurring are rational numbers. That means the numbers which are not rational numbers on the number line are irrational numbers i.e., the numbers which cannot be written in the form of $\frac{a}{b}$ are called irrational numbers. That means the decimal numbers which are non-terminating and non- recurring are irrational numbers.

Ex: 1.01011011101111.....
 5.245246247248.....

d. Real number in different numbers systems :

Consider $\frac{1}{7} = 0.\overline{142857}$. Here the number is in base 10 system. Here 142857 repeats .If we change it into base 7 system

$\frac{1}{7_{(10)}} = 0.1_{(7)}$ i.e., it is terminating number.

So if a rational number in one number system is changed into to another number system, still it is a rational number. But an irrational number in one number system is remains an irrational number in any other number system.

$\sqrt{2} = 1.4142135623731.....$ in base 10 number system
 $= 1.0110101000001001110.....$ in base 7 number system.

i.e $\sqrt{2}$ is an irrational number in any number system.

e. Union :

$\sqrt{2} + 3$ is it a rational number or Irrational number?

$\sqrt{2} + 3 = a$ a rational number then

$\sqrt{2} = a - 3$ a-3 is a rational number (why?)

But $\sqrt{2}$ is an irrational number, therefore $\sqrt{2} + 3$ is an irrational number.

Show that if a is irrational number and b is rational number then a+b, a-b are irrational numbers?

$3\sqrt{2}$ is rational or irrational number?

Let $\sqrt{3} \cdot 2 = x$ is a rational number, then

$\sqrt{2} = \frac{x}{3}$ ($\frac{x}{3}$ is rational number, why?)

But $\sqrt{2}$ is irrational so $3\sqrt{2}$ is an irrational number.

Prove that if a, b are irrational and rational numbers respectively then $a \cdot b, \frac{a}{b}$ are irrational numbers

f. $(\sqrt{a})^2 = a$

$(\sqrt{a})^3 = a\sqrt{a}$, therefore the power of an irrational number is either rational or irrational number.

g. **Why?**

$2^{\sqrt{2}}$ is an irrational number and $(\sqrt{2})^2$ is a rational number. But

$(\sqrt{2^{\sqrt{2}}})^{\sqrt{2}} = 2$ is a rational number.

So the exponent number of a irrational number is irrational or rational number, the number is either rational or irrational number.

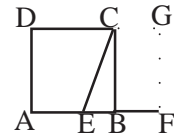
But if we observe $\pi^\pi, 2^\pi, \pi^{\sqrt{\pi}}$ they seem to be irrational numbers. But how to find is not yet found.

h. **h. No tired :**

When we discussed about Fibonacci number series, we discussed that the ratio between two consecutive numbers in the series is near to the value 1.6 and this ratio is called Golden Ratio. And in the chapter "Ratio" we discussed that if the ratio of length and breadth of rectangle is in golden ratio then it seems to be very beautiful. Because of this the painters and engineers would make their paintings or plans in the golden rectangular shape. But how is it to be constructed?

Construct square ABCD with side equal to the breadth of required rectangle. Plot the midpoint of AB and name it as E. Plot F on AB such that EC=EF. Construct AFGD on AF.

AFGD is golden rectangle.



Take $EB = 1$, and show that $\frac{AF}{AB} = \frac{\sqrt{5} + 1}{2}$. This is also can be written as $\phi = \frac{\sqrt{5} + 1}{2}$.

Find its value up to three decimal places. It is an irrational number.

i. In the above golden rectangle AFGD if we remove the square ABCD, then the remaining rectangle is also a golden rectangle. Similarly if we remove the square part from the remaining golden rectangle again, the remaining part (rectangle) is also a golden rectangle. Like this way if we remove the square parts from the golden rectangles then the remaining rectangles are also golden rectangles.

If this process is continued we will get only golden rectangles. So all these are called Dynamic rectangles.

j. **Paper sizes :**

In another context also we can observe the golden rectangles.

The paper which is used in the computer printer is called A_4 size paper. Why?

If the ratio of length and breadth of a rectangular paper whose area is 1 sq.m. is $\sqrt{2}$ then it is called A_0 size paper. If it is halved then the each half is called A_1 size paper. If A_1 size paper is halved then

it is called A_2 size paper. Similarly we will get A_3, A_4 size papers. The ratio of length and breadth of all these different size papers is $\sqrt{2}$. Therefore all these are golden/Dynamic rectangles.

Note that the resulting shapes/sizes in the above two contexts are golden/dynamic rectangles why because the first one is a golden rectangle.

Did you observe anywhere the use of Irrational numbers? -Discuss

k. Adjacent there is no room/space, Are there no neighbours? :

On the number line rational as well as irrational numbers are there. How dense they are means there is no space at all between two numbers i.e., the number line is full of with real numbers, for every number on the number line on both sides there are numbers. That means we cannot say the adjacent number for any number on the number line. Discuss why this is happened?

l. Probability :

In our discussion points this is the last one. On the number line there is infinite number of rational and irrational numbers. If we put sharp edge of a pencil at a point on the number line, then it is too difficult to tell whether the point represents rational number or irrational number.

But the following experiment trays to give answer

Take a dice of 10 faces and write the numbers from 0 to 9 on its faces (one number on each face). Roll the dice, observe the number on its face, write the same number after decimal point (take its integral part as zero). Roll again the dice, observe again the number on the dice, and write this number in the second place. If it is continued, find whether the resulting number is rational number or irrational number.

Indian arts and cultural rangoli are not only beautiful to look at, but there is much mathematics to learn.

- SCF 2011

B) Approach paper on Algebra

Importance :

1. Calendar:

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

- ◆ Take any 3 x 3 square in the above table and find the sum of the numbers in it
- ◆ Find the least number in the same 3 x 3 square
- ◆ Add 8 to the least number and multiply the sum by 9
- ◆ Compare the first and second results
- ◆ Can you explain the secret of this puzzle with the help of Algebra?

2. Pythagorean Triples :

- ◆ Take any two consecutive even numbers (Ex:- 2 and 4)
- ◆ Find the sum of their reciprocals (Ex:- $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$)
- ◆ Consider numerator, denominator, denominator+1 from the above resultant fraction (Ex:- 3,4, 4+1=3,4,5)
- ◆ Check whether the resultant numbers are Pythagorean triples
- ◆ Check whether it is true for any two consecutive even numbers
- ◆ In this process which branch of mathematics is used?

3. Magic Squares :

8	1	6
3	5	7
4	9	2

3 x 3 magic square

4	14	15	1
9	7	6	12
5	11	10	8
16	2	3	13

4 x 4 magic square

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

5 x 5 magic square

Above are 3 x 3, 4 x 4, 5 x 5 magic squares. When these are filled with first 'n' natural numbers i.e., 3 x 3 magic square is filled with 1 to 9 numbers, 4 x 4 magic square is filled with 1 to 16 numbers then the constant (Sum of the numbers in a row/column) are given below

Type of magic square	3 x 3	4 x 4	5 x 5
Constant	15	34	65

From the above table can you guess the constant of 6 x 6 magic square?

Let us observe the above table in detail once again

Type of magic square	3 x 3	4 x 4	5 x 5
Constant	$15 = 3 \times 5$ $= \frac{3 \times 10}{2}$ $= \frac{3(3^2 + 1)}{2}$	$34 = \frac{4 \times 17}{2}$ $= \frac{4(4^2 + 1)}{2}$	$65 = 5 \times 13$ $= \frac{5 \times 26}{2}$ $= \frac{5(5^2 + 1)}{2}$

Hope that you understood the relation in the above pattern. If we proceed in the same way then we will get the constant of 6 x 6 magic square as $\frac{6(6^2 + 1)}{2} = 111$.

Can you generalize this relation with the help of Algebra? And can you check it?

4. Prove the following :

- ◆ Show that the sum of a number of four digits and the number formed by reversing the digits is always divisible by 11.
- ◆ The greatest and least of four consecutive numbers are multiplied together; so also are the middle pair. Show that the difference of the two products is always 2.
- ◆ The sum of two consecutive numbers is always an odd number.
- ◆ Which branch of mathematics is used in the above proofs?

Then do you agree the following :

- 1) Algebra is a powerful branch of mathematics to solve puzzles and problems.
- 2) Algebra is very helpful in examining the rules/laws.
- 3) Algebra is used in generalization of laws/rules.
- 4) Algebra is used to generalize a pattern.
- 5) Algebra is used in finding an unknown value/quantity.

In this way the knowledge and understanding of algebra helps not only to solve the puzzles and daily life problems but also helps for the development of mathematics.

If we don't learn algebra we cannot understand many ideas discussed in Chemistry, Physics, Earth science, Economics, business, Psychology and other areas. Lack of knowledge in Algebra limits our opportunities. Without algebra it is not possible to do many things as easily as at present and it is even not possible to find/discover/invent some results.

If we want to explain anything for one time there is no need of algebra. But we want to explain anything for more than one time (More times) then we need algebra. Algebra is best mathematical language to explain such things very easily. For example if we want to explain how to find the product of two fractions, we have to write in the following way:

"Multiply the numerator of the first fraction with the numerator of the second fraction, and denominator of the first fraction with the denominator of the second fraction, then write the results as numerator and denominator respectively."

But by using the algebra we can write it very easily in the following way:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

It is very handy and it can be understood by each and every one. If we are able to explain the uses of algebra, in this way, then students may get inspired to learn algebra.

Algebra - Application :

- Formulas :** Once a news paper in Chicago, in its sports column published the following news item.

"Competitions will be held within 3 weeks, but the pre-competition game to get all teams ready, which is going to be held on Friday night, is as important as the **formulas** for high school student, so watch it without fail".

From the above sentence we can understand the importance of the formulas in mathematics. Further we can understand that the writer also knows about how far the application of algebra is needed in sports and games. Really speaking many of the issues pertaining to the sports and games have a link with formulas. For example if a player gets T points in n_1 games, then his/her average score is $\frac{T}{n_1}$. Similarly we can estimate a player's individual capacity by taking his/her maximum and minimum score (range) into consideration, further we can calculate the winning percentage of a team with the formula $\frac{W}{W+L} \times 100$, where W = number of wins, L = number of losses.

Similarly to know the area of our residential flat, to know how much cloth is required for our shirts, to know how much length of fence is required to fence our house, to know how much length of ribbon is required to pack a gift that will be given to our friends, in the calculation of rebate, tax, sales tax, income tax and in every moment of our life, we use formulas. We use very simple formulas to calculate them. But all formulas, like above are not so simple. For example to know what date falls on which day the formula is

$$W = d + 2m + \left[\frac{3(m+1)}{5} \right] + y + \left[\frac{y}{4} \right] - \left[\frac{y}{100} \right] + \left[\frac{y}{400} \right] + 2$$

Here D = day of the date
 M = month of the date,
 Y = year of the date

But when we are taking the value of month we have to take the value of January as 13, February as 14, and the value of remaining months as it is i.e., the value of the month march is 3, April is 4 December is 12. And [] means we have to take integral value only

i.e., $[5.2] = 5$, $[14.75] = 14$.

Once "w" is computed divide 'w' by '7' and as per the remainder we find the day of the date. If remainder is zero, the day is Saturday; remainder is 1, the day is Sunday and so on with Friday = 6.

Similarly we can compute the value of monthly installment when the amount "A" is borrowed with rate of interest 'r', for 'm' months from the following formula.

$$P = A x^m \left(\frac{x - 1}{x^m - 1} \right)$$

where $x = 1 + \frac{r}{1200}$ here 'p' is monthly installment.

According to this formula if ₹.8500/- is borrowed with 11.25% rate of interest for 4 years the monthly installment that is to be paid is ₹. 220.72

2) **Functions** : the following are the examples of functions.

How the various health factors will change with respect to age? How the increase in weight affects on health conditions of person,? How a family budget will effect, when their spending habit is changed, how the growth of population effects on use of different resources/ energies on the earth.

Without using algebra/functions, we can also able to explain them with large tables and graphs. But we cannot get as much clarity, simplicity as when algebra is used. Further with the help of algebra we can find when they have maximum value and minimum value. All these are foundations for calculus.

3) **Linear Equations** :

Anything that changes at a constant rate gives rise a linear equation of the form $T = Ax + By$. By using such equations how the real life problems are to be solved is given in the 8th class text book. Similarly how to find the cost of renting a car; the cost of renting a book from a library etc., is given in the new 9th class text book.

4) **Slope** :

Any quantity that changes must change at some rate. That rate of changes is often very important. The change in car's velocity affects its acceleration. The change in our earnings affects our financial status. The change in the status of unemployment will affect inflation. The algebraic idea behind all these notions is "slope".

5) Exponents :

The branch of mathematics which is developed to estimate the possibilities of an incident is 'probability'. Now exponents are being used in this branch. If any quantity that changes at a constant growth rate, that growth is called exponential growth. We can observe such growth in our daily life. Compound interest, mortgage value of a vehicle, payments on credit cards, payments of life insurance policies, benefits on retirement... all are belong to this category. The same mathematical concept is used in predicting growth in population, animals etc.

6) Quadratic expressions :

According to Newton "the path of any object, under the influence of gravity, will be explained by the quadratic expression $Ax^2+Bxy+Cy^2+Dx+Ey+F$. The same is applicable to the stars, planets, comets and moons. Similarly we can note that the same is also applicable to the paths of rockets, bullets, base ball, basket balls. That means this concept is used in physics, space science, armed services, games and sports.

7) Logarithms :

These are used in rector scale used to measure the intensity of earthquake, Ph scale used to measure the acidity, the star magnitude scale used in astronomy.

8) Permutations and combinations :

To predict the result in lotteries, to determine number of people that needed to conduct opinion polls, to predict the T.V. ratings, these permutations and combinations are used.

Like this way algebra is very important branch of mathematics. If we visit a place, but don't know their language, we can any way able to get along by using signs/signals. But we cannot understand their social, political and cultural values and status; we cannot even able to appreciate the richness of their culture. And the pity thing is we even cannot understand what we missed in this visit. Similarly we can learn mathematics without algebra, but we cannot able even to appreciate its beauty, structure and mystery.

History :

generally we feel that arithmetic taught in lower classes is very easy and the algebra taught in higher classes is difficult. The reason, probably, behind it is arithmetic deals with very familiar symbols like digits/numbers, where as algebra deals with unfamiliar symbols like x, y.....But practically there is no difference in using symbols like digits/numbers and symbols like x, y

In earlier day of civilization the following was to be used.

" If you have six fruits and if I give you five more same fruits, then how many fruits do you have all together?"

we know that eleven fruits is answer for this question. i.e.,

six fruits + five fruits = eleven fruits

But if the numbers are very large, writing in the above method is very tedious and time consume. As a remedy for this problems, 1,,2 . . . 9 were developed. These are called numerals.

We already learnt how to write bigger numbers with these numerals. Because of this it is possible to write bigger sentences like "six thousand seven hundred and fifty two" simply as "6752".

By all means you understood the use of signs or symbols. In our daily life we use practically all such symbols for example we use the symbol "horse" to represent a particular life on the earth. Like this way in our daily life to express anything very easily, we use signs/symbols. We learnt all these symbols in our earlier age in lower classes and we got sufficient training. Due to this we felt that all these are very easy. But if we want to use new signs/symbols in higher classes we will hesitate somewhat. We naturally do not like such new symbols to use. But remember that when we were learning the new symbols like 1, 2, . . . 9, +, - in lower classes we would have faced the same problem. Any how it is clear that the use of symbols will simplify our life style.

Observe the following questions.

- 1) If two is added to two then what is sum?
- 2) If five is subtracted from eight what is the result?

By using symbols these two sentences can be written as in the following:

- 1) $2 + 2 =$
- 2) $8 - 5 =$

But when we changed them into symbolic from we would unable to write symbols for total sum in the 1st sentence and for the result in the 2nd sentence. That means we need some other new symbols. But some of us may feel that why we need new symbols, what is wrong if we leave an empty place instead of using symbols. Let us observe the following example:

How much number of mangoes is added by six mangoes to get eleven mangoes? Here we don't know at the beginning how many mangoes are there? This is an unknown quantity. If we write the above sentence by leaving empty space for this unknown quantity, it is like the following:

$$6 = 10 \rightarrow (1)$$

It looks funny and is hard to read. So it is not the better thing to leave blank space for unknown. How the symbol is 6 for six, 10 for ten, and '+' for addition we also need a symbol for unknown quantity.

It is felt that you understood the importance and need of symbols. If we are thinking that using symbols for unknown that means we are thinking about algebra. If a symbol is made for unknown then we will try to use this symbol for all unknowns in arithmetic. That is why algebra is called as generalized arithmetic. Now our problem is to find a symbol for this unknown. For this purpose many symbols were examined. When they were examined some issues were kept in mind. They are, it should be

familiar to all, it should be handy, easy to spell/speak, and it should be least frequently used in its original place. The symbols with such characters are x, y, z Of English alphabet.

By using "x" for unknown in the above equation (1) it seems to be in the following:

$$x + 6 = 10 \rightarrow (2)$$

By observing (1) and (2) we find that (2) is easy to understand. Then

**Do you agree with the following statement?
"Symbols make life simple".**

The first person to use symbols for unknown was a French mathematician named Francois Vieta.

Variable :

Pavani, and Sagar were drawing the models of cars. They use block stickers for the wheels. Pavani made the following model of car with two block stickers.



Then Sagar with the help of two more block stickers made another model of car and put it next to the first model as in the following figure



Mean while their friend Ravi joined them. He asked them "how many block stickers will be required to make four such 4 cars." Immediately Pavani counts the number of block stickers in the above two models doubles the number and says 8.

"Well" Ravi said and asks them, "How many block stickes will be required to make such 59 cars. Pavani and Sagar feel this method of counting stickers Is a bit laborious and time consuming, especially when the number of faces are very large. They decide to find a new way. They think a while and make the following table.

Number of Cars	1	2	3	4	...
Number of required block stickers	2	4	6	8	...

Do you deduct any relation from the above table? The same relation was written by Pavani and sagar in the following way:

$$\text{Number of required block stickers} = 2 \times \text{number of required cars} \rightarrow (1)$$

From this relation Pavani and Sagar are able to find the number of required block stickers to make 59 such care models. To understand easily, if we let the number of required car modes is "x" then

Number of required block stickers = $2x$.

If we want to make a car model then the value of $x = 1$. So the number of required block stickers = 2

Similarly if we want to make two car models then the value of $x = 2$. So the number of required block stickers = $2 \times 2 = 4$

Like this way we can find the number of required block stickers to make the number of required car models.

Here 'x' is called variable. It takes the values 1 or 2 or 3 That means the value of 'x' is not fixed. Its value changes accordingly.

Give some examples of where we use variables in our daily life.

E. Approach paper on Geometry

Importance of study of Geometry :

Geometry is a branch of mathematics concerned with questions of shape, size, relative position of figures and the properties of space. Geometry arose independently in a number of early cultures as a body of practical knowledge concerning lengths, area and volumes, with elements of a formal mathematical science emerging in the west as early as Thales (600 BC). By the 3rd century BC geometry was put into an axiomatic form by Euclid (Greece), whose system considered as Euclidean Geometry set a standards for many centuries to follow. Euclid's collection of geometrical treatise "The elements" had given world wide changes in the study of geometry. This book has been considered as most popular and most translated book into other languages other than Bible.

The study of Geometry laid foundation to the entire educational system in western countries and created logical thinking process among you this from Euclidean time. Its study has been sidened to solve many problems after non-Euclidean geometry came into exist. Scientists, Engineers used many geometrical applications to find new relations and developed wonderful constructions like dams, bridges, roads etc. The art and culture has been developed further and painters, designers and skilled persons created wonderful arts and designs through 3D figures.

For example, the present cinema screen, TV screen were naturally the application of Euclidean geometry only. But it is unfortunate that this kind of applications were not given importance in secondary mathematics curriculum.

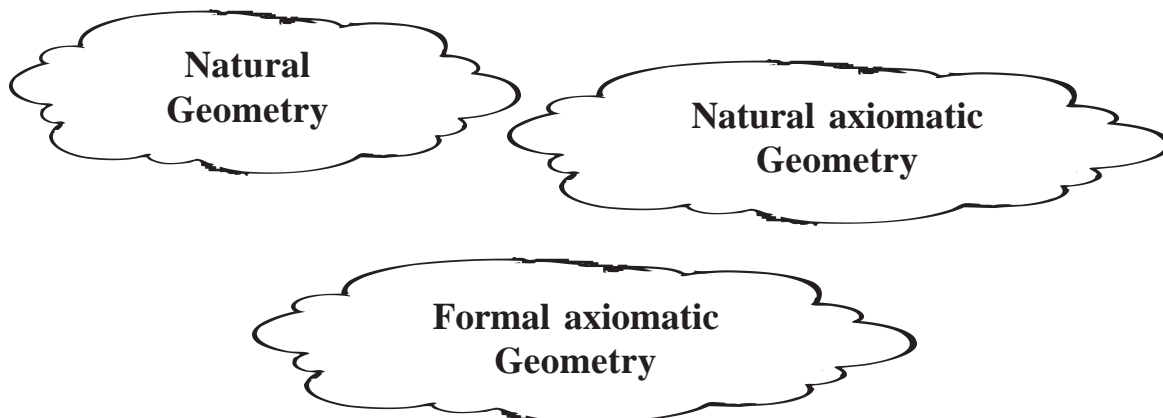
Geometry is derived from two Greek words "Geo" means earth and "metron" means measurement. Ancient Greeks arranged the concepts and axioms according to the logical order. There are certain evidences about circumference of a circle is divided into 360 equal points by Babylonians, right angled triangle theorem is used before Pythagoras and ratio proportional problems, properties of triangles and relation between sides and angles of triangle.

The study of Geometry gives many answers to question raised from Babylonians to Egyptians. All these questions were begin with "How?" For example, how can you find the interest on an amount borrowed? How can we calculate the volume of a pyramid? How can we find the square root of a given number etc were given for reasoning raised many logical thoughts. The importance of Economics increased since 100AD in Western and middle East lay the foundation to learn mathematical language. The mathematical questions were changed according to the new thinking process and begin with "Why". For example why the area of triangle is half the product of their base and height? 'Why' the square on the hypotenuse is equal to the squares of sum of two sides

of right angled triangle? This type of questions promoted many people to think logically with reasoning. The revolutionary approach leads to many developments in Science and thechnology and Trade and comerce. Though the study of geometry undergone many changes the foundations laid by Greeks still influences the society.

How to learn Geometry? :

We can divide the learning of Geometry in three stages.



"**Natural Geometry**" is intimately related to reality and reasoning is close to experience and intuition. The objects of this geometry are material objects, graphic livges on a paper sheet or virtual lines on a computer screen. Drawing and measurement techniques with oridinary geometrical tools (rules, set square, compass) as well as experimentation in the sensible world (using techniques such as paper folding, superposing) are used in this stage. New knowledge may be produced based on evidence, experience or reasoning, while a permanent motion between the model and the reality enables the student to 'prove' the assertions. The kind of experiences were given in 6 and 7th class new text books.

"**Natural axiomatic Geometry**" deals with geometrical figures in which the new relations were obtained through investigations. The objects are ideal, so reasoning relies on the mathematical properties of the abstract geometric objects. a system of definitions and axioms is necessary for the creation of the objects. In this system the axioms are as close as possible to intivition, but making progress and reaching certainty demands demonstrations inside the system. Hypothetical deductive laws are the source of validation. The topics covered in class 9th and 10th were based on certain geometrical relations obtained by the sutdents through verification and activities. For example, to prove "The diagonal divides the parallelogram into two congruent triangles", the child should verify the fact by drawing a parallelogram with suitable measurements and by cutting it into two pents through diagonal and verify whether they coincide eachother or not. He should also know certain axious related to congruency of triangles. this leads the child to prove the theorem using axiomatic approach.

"**Formal axiomatic Geometry**" discuss the concrete ideas of geometry in symbolic form. The lines, curves, triangles, circles will be translated as algebraic equations. This leads to discover new relations in engineering and Technology. For example $ax + b = c$ ($a \neq 0$) is a linear equation in one variable, which represents a straight line whereas the quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) represents a curve.

The passage from natural Geometry to axiomatic natural geometry is a complex, sensitive and crucial matter. Since these two paradigms are different as far as objects, techniques and validation mode are concerned. Moving from Natural Geometry to Natural axiomatic Geometry students have to change their theory concerning the nature of the objects and of the space. They are forced to adopt the notion of conceptual objects, the existence of which is based on a definition in an axiomatic system. Consequently, they have to foster new techniques to work relying on the mathematical properties of each abstract geometrical figure. For example, as a consequence of the continuous practice of the pythagoras' theorem and the didactical contract formed during teaching, students consider that they are expected to apply pythagoras' theorem anytime a right angle is involved in a geometrical figure.

Discussion points

Discuss and write.

- 1) Make a list of some objects you will find at home and write geometrical concepts involved in them.
- 2) Give two examples each in which children learn Geometrical concepts through (a) objects (b) mathematical instruments (c) Experiments (d) logical reasons.
- 3) Explain method of proving the statement "Sum of three angles of a triangle is 180° ".

Geometrical constructions - A Special approach :

Geometrical constructions play a vital role in the Study of Geometry. These constructions will help the children to understand the accuracy and logic behind the geometrical figures. It is a very interesting feeling of persons of different age groups while drawing or constructing figures. It is a challenge to the students. If they learn the basic geometrical constructions in the beginning, it will certainly help them to construct difficult constructions later.

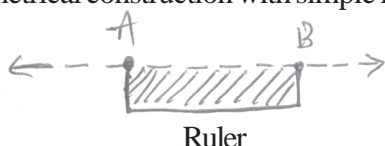
Practicing the basic geometrical constructions and using them in meaningful situations, using proper language to express the construction, choosing proper instruments will lay a foundation to understand abstract concepts behind Geometry. These are the pillars of construction of knowledge in Geometry.

In the development of Geometrical constructions there is a need of Justification through axioms at secondary level. For any basic geometrical constructions, there is a rule to use two primary instruments a ruler and a compass only. But recently we are using mostly a graded scale to draw line segments and a protractor to construct angles. Though these instruments will help the children to do

the construction with ease, there we are neglecting the national inner abilities of the children, and hence they are not enjoying the beauty of geometrical constructions.

'Ruler' is used to draw a line segment between two given points and 'compass' is used to draw an arc or circle when a fixed point and given a radius. Some people argue that all line segments and angles can not be made by the two instruments.

We can not trisect all angles, but Greek people used these two primary instruments to prove any geometrical construction with simple logic.



Ruler

Compass

Generally, we observe two types of questions appear in text books in Geometrical constructions

- 1) Draw the figure using the given measurements
- 2) Construct the figure using the given measurements.

- ◆ Here for drawing Geometrical figures, we can use any available instruments, whereas to construct the figure, we have to use ruler and compass only.
- ◆ We can see the geometrical constructions given in the new textbooks were different from earlier books. To make the children understand better way of constructing the geometrical figure, figures were given for each step and motivated the children to use ruler and compass only. The children have to use protractor for verification of the angle in the figure and not for constructing an specific angle. The systematic algorithms given in new textbooks will help the children to understand the logical reasoning behind the constructions.

Discussion Points

Discuss and write.

- 1) Construct an equilateral triangle using ruler and compass. Write steps of construction.
- 2) Construct the following angles using ruler and compass and verify using protractor.

a) 45°	b) 90°	c) 135°	d) 180°
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- 3) Explain how you can trisect 90° angle.
- 4) Construct and verify the Euclid's parallel axiom (Modified by John Playfair). "Through a point not on a given line, exactly one parallel line may be drawn to the given line" using ruler and compass.

Reference Books :

- The Geometry reasoning of Primary and Secondary school students - George panoura, Dept. of Education, University of Cyprus.
- Position paper "Teaching of Mathematics" National curriculum Framework 2005, New Delhi.
- Position paper "Teaching of Mathematics" State Curriculum framework-2011, A.P.

F) Approach Paper on Statistics

What is Statistics?

Statistics is a branch of Mathematics which deals about collection of information, scientific analysis of the data, interpreting and drawing inferences from the data, making estimations, . . etc. It also discusses about the pros and cons of previous projects and makes future plans.

Why do we study Statistics?

In every field of life, such as business, medicine, scientific research, administration, economics, . . etc., it became a routine to collect, analyse and interpret the numeric data, compare and contrast two similar data and to plan for future projects. On par with scientific advancement and population growth, it is inevitable to collect and analyse the numeric data. Many intellectuals are required to work upon. Here statistics helps to fulfil the needs of the globe.

Every individual is benefited when he / she study numeric data and able to make out inferences by scientific reasoning. Every information centre of every department exhibits the information in the form of tables and graphs. An individual who have the knowledge of these can only be benefited. So it is inevitable for every social being to learn statistics.

The sexy job in the next ten years will be statisticians. Because now we really do have essentially free and ubiquitous data and the complimentary scarce factor is the ability to understand that data and extract value from it.

--

Hal Varian, Google's chief economist(2009)

Evolution and History of Statistics :

Many believes that statistics has evolved as a subject in 1662, when the book, 'Natural and Political Observation upon the Bills of Mortality', written by John Graunt was published first. Initially scope of statistics increased with the use of statistical methods to analyse the religious, demographic and economical data. Now it is not a wonder that there is no field in the world, which is not using the statistics for its progress. Development of computerisation made the usage of statistics intensive.

What are we going to study in statistics during Secondary Education?

On the basis of level of comprehension of the children, the following are discussed step by step from VI to X classes.

- Collection of small Data
- Organisation of Data
- Interpretation and representation of data in terms of Picto graphs, Bar graphs, pie charts, . . etc.
- Constructing Frequency Distribution tables
- Determining the values of Central Tendency
- Drawing inferences and Commenting on Data.

What is a Data? Why it should be organised?

Collection of numeric or any other information, collected in direct survey for a particular purpose or collected from some other data, is called 'Data'

If there are limited number of observations, it is easy to analyse and interpret the data, but if there are many number of observations it is difficult. Then interpretation can be made easy by organising the data in a table form or by constructing frequency distribution. This makes the data concise and easy to handle, but interpretation becomes approximate.

Example 1:

Primary data : Means of transport in each of students houses are listed as

Anitha Auto

Bharathi bullock cart

Chandrasekhar bicycle

Damodar scooter, bicycle

Fareed bullock cart, bicycle

Giridhar bike, auto

Peter bike

Sanjana none

Salma bicycle, auto

Umesh bullock cart

Organised data :

		Means of transport	Number available
		Bullock carts	3
		Bicycles	4
		Scooters	1
		Bikes	2
		Autos	3

When there are limited number of observations, recording the observations, comparing themselves and comparing with other data and interpreting the inferences is simple, but when the observations are many it is possible only when frequency distribution is constructed.

Ex: Consider the marks of 50 students in Mathematics in Quarterly examination

Primary data :

31, 14, 0, 12, 20, 23, 26, 36
 33, 41, 37, 25, 22, 14, 3, 25
 27, 34, 38, 43, 32, 22, 28, 18
 7, 21, 20, 35, 36, 45, 9, 19
 29, 25, 33, 47, 35, 38, 25, 34
 38, 24, 39, 1, 10, 24, 27, 25
 18, 8

Class Intervals (Marks)	Tally Marks	Frequency (No of students)
0`7		4
8`15		6
16`23		9
24`31		13
32`39		14
40`47		4

Ascending order of the data:

0, 1, 3, 7, 8, 9, 10, 12, 14, 14, 18, 18, 19,
 20, 20, 21, 22, 22, 23, 24, 24, 25, 25, 25, 25, 26,
 27, 27, 28, 29, 31, 32, 33, 33, 34, 34, 35, 35, 36, 36,
 37, 38, 38, 38, 39, 41, 43, 45, 47.

Grouped frequency distribution tables is shown aside

If we observe the above three forms Frequency distribution seems to be more concise and comprehensive. So we practically use the frequency distribution of a data.

Let us find out how easy is to interpret a data of limited observations.

Example 3: Price(in rupees) of a kilogram of rice at different shops is 22, 35, 37, 42, 24, 35, 41, 37, 40, 37, 30, 39, 42, 38, 37, 41, 37.

Ascending order of prices is : 22, 24, 30, 35, 35, 37, 37, 37, 37, 37, 37, 38, 39, 40, 41, 41, 42, 42.

- ◆ Minimum value of observations is 22; Maximum value is 42; difference of these is $42-22=20$. This is called 'Range' of the data. It represents the spread of observations in the data.
- ◆ Out of 17 observations, the middle most observation is 'Price of a kilogram of rice is ₹ 37'. It means in how many shops the price of rice is more than ₹ 37, in same number of shops the price is higher than ₹ 37. This value is called 'Median' of the data.
- ◆ By observing the data we can find that ₹ 37 repeats for more number of times. It means in many number of shops price of kilogram of rice is ₹ 37. This is called 'Mode' of the data.
- ◆ Similarly by dividing the sum of observations by number of observations to get 'Average' or 'Arithmetic mean' of the data.
- ◆ Among the above Average / Arithmetic mean, Median, Mode lie at the centre of organised data and so these are called 'Values of Central Tendency'.

When to use which value of Central Tendency?

In classes VI and VII, determination of values of central tendency is discussed comprehensively. But a particular value of central tendency is required to interpret a particular data. This is to be taken care while explaining and drilling. More exercises are given in VII class for the purpose.

How to Construct a Frequency Distribution Table?

Out of many observations of a data, if there are few distinct observations, then the data can be organised in frequency distribution table (as in example 1). This is called 'Ungrouped Frequency Distribution'.

If there are too many distinct observations, ungrouped frequency distribution of the data cannot be concise or easy to read. Then ascending or descending order of observations can be divided into convenient groups and observations can be noted as frequencies. This is called 'Grouped Frequency Distribution'. When to use Ungrouped of Grouped data is discussed with examples in VIII class. Make distinction.

Determination of values of central tendency of (i) primary data is discussed in VII class and (ii) ungrouped frequency distributions is discussed in IX, where as (iii) grouped frequency distributions is discussed in X class.

Graphs for Comprehensive Understanding:

Graphical representation of a data is more understandable rather than in tabular form. When the observations of data are not inter-dependent and distinct observations are few with frequencies we use to represent the data with pictograph. These can be interpreted even by small children and illiterates. So we use them right from the beginning of study of statistics, i.e., from III class onwards. But construction of these is time consuming and so, from VI class onwards, we introduce 'Bar graphs' instead.

In bar graphs different observations are represented by bars of equal width and lengths proportional to their frequencies. As all the distinct observations are independent from one another series of bars can be altered for convenience. When the frequencies of different observations are differ by small extent the data is shown in Vertical Bar Graph and when they differ by much Horizontal Bar Graph is used (you may guess 'why?')

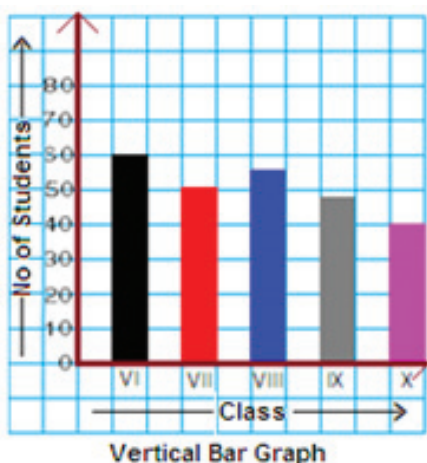


Fig. 3

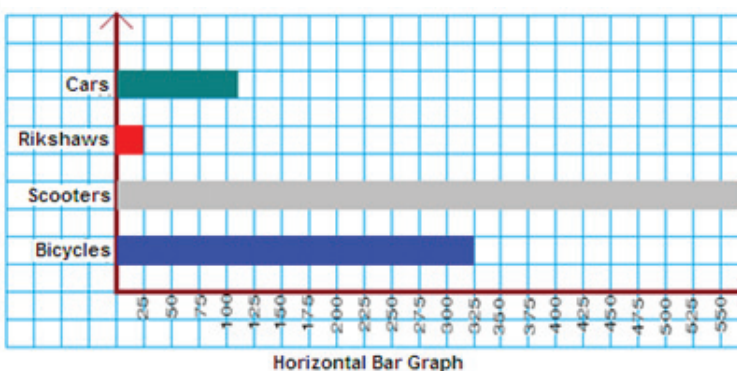


Fig. 4

Which of the following data are to be shown as vertical bar graphs and which are horizontal bar graphs? Discuss.

- 1) Data about the number of boys, girls, youth, old aged people in a colony.
- 2) Stock in a fair price shop
- 3) Production of different brands of cars in an year in India.
- 4) Ingredients used in preparation of a South Indian Dish.
- 5) Rainfall in a place during successive years.

As the understanding level of children is increasing 'Double Bar Graph' is introduced in VII class. If all the distinct observations of a data are considered as parts of something and they are interdependent 'Pie Chart' is used. In this chart angle of each sector is proportional to fraction of each observation in the whole, keeping the sum of the angles is 360o.

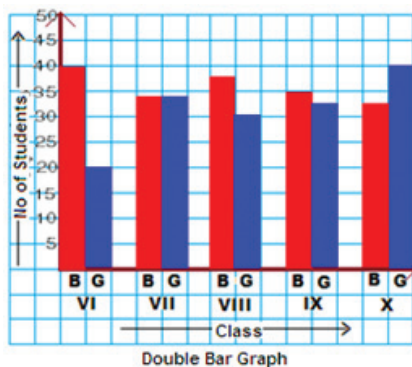


Fig. 5



Fig.6

What is the difference between a Bar graph and a Pie Chart?

In a bar graph lengths of bars are compared to interpret the data, but lengths of different bars are not inter dependent. In case of a pie chart, each sector represents an observation as how much part of the data it is. (Area of each sector is to area of the circle is as to the quantity of the observation to the whole data). Change in the quantity of every observation affects the fraction of the other observations.

Which of the following data are to be shown as bar graphs and which are pie charts? Discuss.

- 1) Yield of different crops in a village.
- 2) Achievement of a student in different subjects.
- 3) Allocation of funds to different expenditures from the salary of person (Monthly budget of a salaried employee)
- 4) Allocation of funds to different fields in Andhra Pradesh State Budget.
- 5) Number of students of each class in a school.

Up to this point we have discussed about graphical representation of raw data or a data of observations with frequencies (ungrouped frequency distributions). Now we discuss about the graphs of grouped frequency distributions.

Graphs to draw inferences from a data :

To represent grouped frequency distributions we use graphs like 'Histogram', 'Frequency Polygon', 'Frequency Curve, etc. Though the data is same represented in the above three graphs with class

intervals / mid values on X-axis and corresponding frequencies on Y-axis, successively their accuracy increases in drawing inferences. This is explained in VIII class.

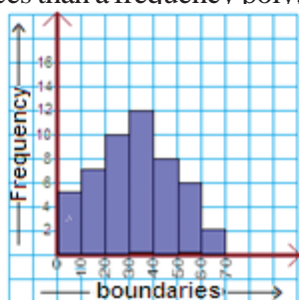
In histogram all the vertical bars similar observations in different intervals and bars are touching one another. As the bars represent the frequencies in successive intervals position of bars should not be altered. Area of all the bars represent the number of observations in the data. This graph is useful in finding the mode of the data.

In frequency polygon also area sirscribed by the polygon with X-axis represents the number of observations in the data. It is useful to estimate the frequency of any observation in the data.

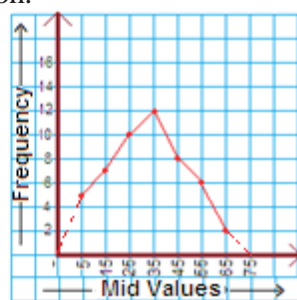
Frequency curve is also useful to estimate the frequency of any observation in the data. But it gives more accurate inferences than a frequency polygon.

Class Intervals	Frequency
0 – 10	5
10 – 20	7
20 – 30	10
30 – 40	12
40 – 50	8
50 – 60	6
60 – 70	2

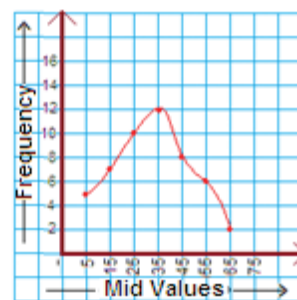
Frequency Distribution



Histogram



Frequency Polygon



Frequency Curve

Fig. 7

Fig. 8

Fig. 9

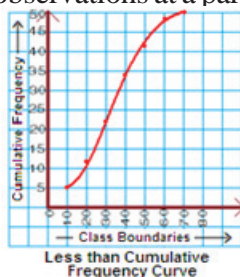
Fig. 10

Other types of graphs used to represent grouped frequency distribution are 'Ozive Curves' (Less than cumulative and greater than cumulative curves). These graphs are useful to find out number of observations of the data which are greater than or less than a particular observation. Slope of these curves represents the rate of rise or fall of observations at a particular observation.

Class Intervals	Frequency	Less than Cum frequency	Greater than Cum frequency
0 – 10	5	5	50
10 – 20	7	12	45
20 – 30	10	22	38
30 – 40	12	34	28
40 – 50	8	42	16
50 – 60	6	48	8
60 – 70	2	50	2

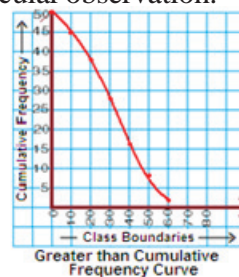
Cumulative Frequency Distribution

Fig.11



Less than Cumulative Frequency Curve

Fig.12



Greater than Cumulative Frequency Curve

Fig.13

When both the ozive curves of a data are plotted on the same graph their intersection represents the Median of the data. All about the graphs is comprehensively discussed in VIII class. Teachers must reinforce these ideas in IX and X classes.

Note :

There are some exceptions in statistics. For example, values of central tendency of a raw data and its frequency distribution are slightly different, mode of frequency distribution is supposed to lie in the class interval of the higher frequency, but it may not be actually. Use of all such anomalies are discussed the text books. For additional details go through the text books and make every student to read the text books.

G. Approach paper on Mensuration

The primary needs of human life are food, cloth and shelter. To find the measure of these needs we require basic measurements. To find the volume of vessel to store the food, to find the area of place required to construct a house, to find the length of cloth for making a dress we need the concept of area and volume. The topics related to these areas are kept as lessons in 7,8 and 9th classes in our new textbooks. We have introduced the concepts and various problem solving regarding areas of rectangle, square, triangle and circle in 7th class. Number of examples were given related to daily life situations in which we require area and perimeter.

We have introduced "Areas of plane figures" in class 8th as a chapter in which Wide variety of problems given to find the area of land required to construct a house when the land is in the shape of a trapezium. Through these examples the formula to find the area of trapezium is derived. In the same way we got this formula by an activity in which a trapezium was drawn on a graph sheet and folding along the diagonal to create triangles. As we know the area of triangle in earlier class, we got the area of trapezium by adding the areas of two triangles. Here you can observe that we have followed the learning principle "known to un known". Another way to find the area of trapezium was also given by dividing the trapezium in to a rectangle and two triangles.(see Area of play ground problem).In all these three cases we have generalised the principle to find the area of trapezium. To reinforce the concept and formula we have been given some illustrated examples.

Activities were provided to children to find area of quadrilateral by triangulation method under Try these as the children were exposed to find the area of any quadrilateral by the previous activities. Some problems were also given to find area of rhombus by giving examples from daily life situations.

Many villagers doesn't know how the surveyors measure the lands which are in different shapes. A surveyor measures the lands using the concept trapezium, rectangle, triangle.

A circle is not made up of line segments. we have used a graph paper to find it's area. But it doesn't give exact area. so an activity was given to find the area of circle by dividing it in to 8 equal parts in which each part as a sector and arranged in an order to get a shape of a rectangle or a parallelogram..But

to find the area more accurately we need to divide it in to 64 equal sectors. This formula has been derived by doing an activity given in the book "A book of juice". We have also provided the topics like area of semi circle, area of rectangular paths with illustrated examples to practice for competitive examinations at national level.

We have introduced the surface area and volumes of cube and cuboid through 2d and 3d figures. As the children were more exposed to 3D and 2D geometrical figures and relation between them topics were given to find surface area of cylinder, cone and sphere in this chapter. Children were provided with activities to find the surface area and volume of these objects in detailed manner. Raised questions to think and discuss how the capacity is different from volume of a vessel. The volume of cylinder is derived through area of a circle(base) and multiplied with it's corresponding height. Relation was established between the volume of cone and a cylinder by an experiment as the volume of cone is $1/3^{\text{rd}}$ of volume of cylinder. The volume of sphere was derived through the volume of pyramid.(see the figure and activity given in the textbook)

Children were provided with more conceptual ideas through logical questions. How a prism is different with pyramid? Are cube and cuboid are prisms? Why a cylinder is not a prism? How a sphere is divided with finite no. of pyramids? Many problems were solved in examples so that the children understand different situations in which the surface areas and volumes are necessary for 3D objects.

Problems in the exercises were given related to daily life and which connects to apply previous knowledge. The academic standard prescribed for this chapter was taken in to consideration in setting up the problems. Problems relating to logical reasoning, communication, representation, connection besides problems were given through out the chapter to assess the children formatively.

Constructive approach was followed to make the children to understand various concepts in finding surface areas and volumes. For example to find the volume of cone, we begin with the objects in conical shape in the nature such as carrot, ice cream cone etc to give an idea of the shape. The formula to find the lateral surface area of cone i.e. $A = \pi r l$, Total surface area $A = \pi r(r+l)$ and volume of cone $V = 1/3^{\text{rd}} \pi r^2 h$ were derived by the basic concepts of area of circle, area of sector and volume of cylinder. The L.S.A of cone is explained by an activity and algorithm. T.S.A of cone is obtained by adding area of the base i.e. circle and L.S.A of cone. By comparison of a cylinder and a cone of same

radius and same height and through a simple experiment the volume of the cone is concluded as it will be $\frac{1}{3}$ rd volume of cylinder. Similarly the surface area and volume of sphere was also derived in a systematic way in which logical reasoning is involved.

Mensuration should always be learned through daily life situations. Every concept in these chapters is carefully taken, promoted the children participation for better understanding. Exercises like do this, try these besides think and discuss were provided to assess the children throughout the chapter to know the skills and application of knowledge in different situations.

The learning environment in the school must also give space to the whole range of mathematical processes- problem solving, use of heuristics, estimation and approximation, use of patterns, visualization, representation, reasoning and proof, making connections and mathematical communication.

- SCF 2011

H. MENSURATION-Approach to a chapter

◆ Introduction :

Observation and identification of 2D and 3D objects in groups

◆ Surface area of cuboid :

- ◆ To derive the formula to find the lateral surface area and total surface area
- ◆ Finding the lateral surface area and total surface through an activity

◆ Volume of cuboid :

- ◆ Explaining the concept through an activity.
 - ◆ Understanding the capacity of a vessel
 - ◆ Deriving the formula to find the volume of cuboid
 - ◆ Deriving the formula to find the volume of cube through volume of cuboid
 - ◆ Finding the volume of right prism by the volume of cuboid and cube.
 - ◆ Deriving the formula to find volume of pyramid by activity.
- ◆ Refer textbook for the presentation of the chapter which includes surface areas and volumes of cuboid, cube and pyramid

◆ Right circular cylinder :

- ◆ Concept of cylinder through an activity.
 - ◆ Identification of right circular cylinders and other cylinders.
 - ◆ Deriving the formula to find lateral surface area of cylinder-do this exercise
 - ◆ Deriving the formula to find total surface area of cylinder-do this exercise
 - ◆ Finding the formula to find volume of cylinder through an activity.
 - ◆ Understanding the concepts and application of knowledge through illustrative examples.
- ◆ (Refer the textbook exercises related to the above concepts)

◆ **Right circular cone :**

- ◆ Concept of cone through an activity
- ◆ Deriving the lateral surface area of cone by an activity.
- ◆ Deriving the total surface area of cone by an activity
- ◆ Deriving the volume of cone through the volume of cylinder.

◆ (refer textbook exercises related to the above topics)

◆ **Sphere :**

- ◆ Concept of sphere through the objects obtained in surroundings
- ◆ Deriving the formula of surface area of sphere through an activity.
- ◆ Finding the lateral surface area and total surface area of hemisphere.
- ◆ Deriving the volume of sphere through an activity.

◆ (refer textbook exercise related to the above topic)

◆ Revision

Revise the concepts of prism, cube, cuboid , cylinder , cone and sphere through What we have discussed.

The Mathematics that child learns to do inside and outside school remaining separate and disconnected. Of course, the larger issue here is of the relation between the school curriculum, and life outside school. Since Mathematics is an abstract branch of knowledge, one may think that there is little to be said about its connecting with culture and everyday life.

- SCF 2011

CHAPTER-6

Teacher Preparation

Teaching learning process is an important aspect of teaching a subject. It includes teacher preparation, presentation and evaluation. Now the curriculum and new text books provides an opportunity to modify the teaching learning process to reach each and every child of the class. Thus the teachers has to modify their teaching strategies in accordance with the prescribed academic standards at each level.

- ◆ Preparation is a vital stage to plan and visualise the teaching learning process in the class room.
- ◆ How to make children to participate in the teaching learning process is a very important aspect of preparation.
- ◆ Assessment of children through continuous comprehensive evaluation is another aspect of preparation .

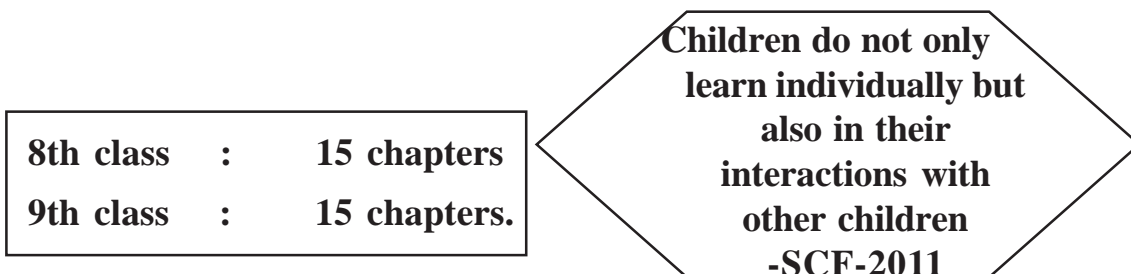
THINK AND DISCUSS :

- ◆ **Is the teaching learning process natural or artificial ?**
- ◆ **Is mathematics teaching taking place in accordance with nature of maths and nature of the child ?**
- ◆ **Do all the teachers know the changes taking place in teaching learning processes?**
- ◆ **Do the children are learning of maths only through teacher ?**
- ◆ **Do the children are learning mathematical concepts through their experiences ?**
- ◆ **What kind of changes do we expect from children in teaching maths ?**
- ◆ **Do the children applying their mathematical knowledge in solving problems in real life situations ? If not , why they are not able to apply ?**
- ◆ **Why people are saying that mathematical standards are decreasing in the days ? Is it a fault of the curriculum or teaching learning processes ?**
- ◆ **Is it a fault of children ?**

The Andhra Pradesh state curriculum framework-2011 has reviewed all the above aspects and made recommendations in the revision of syllabus, suggestions to improve the teaching learning process and evaluation. In this context the syllabus is changed and new textbooks were prepared in accordance with national syllabus. Change is the order of the day. It is necessary that every teacher should understand the changes occurred in the society and process of teaching so as to make every child to participate in learning. So the teachers should revisit their teaching strategies and make necessary changes to improve the quality of mathematical education.

Changes takes place in text books : Let us go through what changes takes place in revising the textbooks once before how to change our teaching strategies.

The syllabus prescribed to 8th and 9th classes is divided in to various chapters keeping in the view to cover all areas. i.e.; arithmetic, algebra, geometry, mensuration, statistics and number system.



Some features of new textbooks

- ◆ The chapters belonged to different areas were mixed together in the annual plan according to CCE
- ◆ Introduced every chapter by a discussion or a dialogue keeping in the mind of children's previous knowledge and daily life experiences.
- ◆ Every chapter is divided in to sub topics and encouraged children participation by discussion method.
- ◆ To assess the child at each sub topic a small exercise under the title "do this" is given. This will help the child too to assess himself on a particular topic.
- ◆ To encourage the children to think logically and give reason to his/her answer some questions under "try this" and "think and discuss" were given.
- ◆ Teaching learning material has an important role in teaching learning process. It will the teacher to motivate and for interaction.
- ◆ Most of the topics in the chapters are meant to encourage the children to discuss with their friends and in groups.
- ◆ The additional topics given in the new textbooks such as Highlight from history, Brain teasers, Do you know etc; will promote the children to develop positive attitude and interest towards learning of mathematics.

A teacher can see the greater shift in his/her teaching in the class using these new textbooks. So every teacher must go through the content included and method adopted in these new textbooks. Every teacher has to plan activities and concept mapping questions before every lesson.

Teacher should act as a facilitator and as a guide to encourage children to participate in the discussion and activities. Also encourage them to learn maths with joy.

THINK AND DISCUSS :

What kind of teaching learning process is required to adopt in the classroom with these new textbooks ? Compare with the process that you are following now.

Steps in teaching learning process

- ◆ Motivating the children with previous knowledge/ daily life experiences/ discussion with introduction given in the textbook.
- ◆ Make the children to understand the concepts by incidents/reading the examples/solving problems.
- ◆ Make them to participate in group discussions and activities for concluding the facts.
- ◆ Ask them to solve the problems in "do this" exercise individually and "try these" questions in groups.
- ◆ Provide situations to identify the errors committed by the children on their own and make them to correct mistakes.
- ◆ Assess the children's performance time to time and record in CCE register.
- ◆ Motivate the children to solve the problems given in the exercises on their own. Help them if necessary.
- ◆ Create new problems to encourage fast learners to solve and find new relations.

"Teaching learning process is a result of over all discussion takes place between two interrelated corresponding axioms"

-Bernand russels

Self reflections:

- ◆ Explain the steps of teaching a lesson from 8th or 9th class mathematics.
- ◆ Prepare a table of activities (individual/group/whole class) for a lesson from 8th or 9th class mathematics.

ROLE AND RESPONSIBILITY OF A TEACHER

- ◆ How a teacher is to be prepared before taking the class?
- ◆ What kind of material he/she has to prepare ?
- ◆ What kind of activities are to be designed to motivate all children to participate in learning process?
- ◆ What are the activities to be planned at the end of class room teaching?

Any kind of textbook is not an alternative to the classroom teacher. The teacher should keep the objectives (academic standards) in mind to plan the teaching learning process.

Let us understand some necessary steps and material to plan the process:

Teacher preparation

1. Preparation : Teaching :

- ◆ Should list out the academic standards of the lesson.
- ◆ Read the chapter thoroughly and solve the problems given in the exercises.
- ◆ Plan individual /group activities are to be given to the children.
- ◆ Plan activities for A, B, C group children as per the content.
- ◆ List out various daily life examples to connect with the topics.

2. Preparation:Teaching learning material :

- ◆ Teaching learning material mostly includes classroom interaction material such as charts, sketch pens, grid paper, graph paper .
- ◆ While teaching statistics, geometry, symmetry, required colourful pictures must be collected from magazines.

◆ Here some material required for every math teacher is listed.

- | | |
|--------------------------|-------------------------------------|
| 1. charts -white, colour | 9. colour papers |
| 2. geometry box | 10. scissor |
| 3. isometric sheets | 11. long scale(1 m) |
| 4. grid paper | 12. graph board |
| 5. graph paper | 13. mirror |
| 6. dice | 14. rope of uniform width |
| 7. coins | 15. 3d objects (cube, cuboid etc;) |
| 8. trace paper | 16. procircle. |

3. Preparation:Time management :

- ◆ Plan to children activities in the allotted time.
- ◆ See that net time is used to involve every child in learning.
- ◆ Plan the time for motivation, presentation and discussion, assessment
- ◆ See ample time given for problem solving.

4. RESPONSIBILITIES OF A TEACHER :

- ◆ Going to class in time and come back only after the allotted time.
- ◆ See all the children to participate in the process of teaching.

- ◆ Allotting additional activities or problems to intelligent children.
- ◆ Record the formative assessment records time to time.
- ◆ Writing teacher dairy and updating it by the inclusion of reflections.
- ◆ Checking the notebooks of the children to make them understand the mistakes they made.
- ◆ Follow formative assessment procedure to record the children progress.
- ◆ Prepare question papers keeping the academic standards in the mind.
- ◆ Plan the activities/more problem worksheets for backward children.
- ◆ Discuss the progress of the children with parents time to time.
- ◆ Discuss issues with colleagues other teachers to get more clarity in content and methodology.
- ◆ Go through reference books, internet for more conceptual clarity.

Self reflections :

How can i plan and prepare to teach the chapter "probability" for 9th class children?

Inadequate teacher preparation and support acts negatively on all of school Mathematics. Generally a teacher teaches how he learned from his teacher. Due to many pressures teachers adopt short cuts to achieve better marks and ranks. Learning achieved in this way is very short lived.

- APSCF-2011.

Chapter - 7

Continuous Comprehensive Evaluation - Understanding

Introduction

In order to identify 'How students are participating in learning process?', 'To what extent students are comprehending on concepts?', 'whether the teaching methods are successful in achieving the academic standards of not?' and to determine the goals of education 'Comprehensive Evaluation' is necessary. Now, in our schools, this is being carried out in two ways, 1. Formative Evaluation and 2. Summative Evaluation. In this chapter we discuss about the procedure of conducting evaluation and how to prepare question papers suitably.

- ◆ **What to be evaluated in Mathematics? How?**
- ◆ **What aspects must be taken care in Formative Evaluation?**
- ◆ **What aspects must be taken care in Summative Evaluation?**
- ◆ **How to identify and consider the mistakes done by the students? How to make use of this knowledge in teaching process?**
- ◆ **What are the principles to keep in mind while preparing a FA / SA question paper?**

What to be evaluated in Mathematics?

Mathematics teaching has the following objectives -

1. Children should be able to develop skill and understanding in the various curricular areas related to number and space.
2. Children should be able to reason mathematically.
3. Children should be able to pursue assumptions to their logical conclusion
4. Children should be able to handle abstraction.
5. Children should develop problem solving abilities- understand/formulate problems, develop a variety of strategies to solve them, verify and interpret results, generalize to new situations)
6. Children should develop confidence in using Mathematics meaningfully.

By consolidating all these, the main objective of evaluation in mathematics is to determine the following competencies.

1. Problem Solving
2. Reasoning and Proof
3. Communication
4. Connection
5. Representation and Visualisation

Note : Clear statements about what students must know and be able to do are called '**Academic standards**'.

How to conduct Evaluation Process?

As a part of evaluation 'Formative and Summative Evaluations' should be conducted, simultaneously with teaching learning process, to measure the level of achievement of the child.

Formative Evaluation :

Formative Assessment is a assessment conducted during the process of teaching. Some of the main features of formative assessment are i. It is diagnostic and remedial and in the form oral, written and performance, ii. Enables teaches to adjust teaching to take account of the results of assessment, iii. Builds on student's prior knowledge and experience in designing what is taught and iv. Encourages students to understand the criteria that will be used to judge their work. The teacher observes and record the children progress and as well as learning gaps through the following.

(1) Participation and Reflections of the students during teaching learning process.(2) Class work, Homework, Portfolio's, Assignments etc., (3) Slip Test (4)Children Projects.

Formative Assessment is Assessment for Learning and assessment as learning. It is through observation of student responses, student engagement, student notebooks, assignments and other written works. Formative Assessments will be conducted by the teacher during instructions. We shall discuss the above tools in detail.

1. Participation and Reflections of the students :

Teacher must observe the students how they are participating in the learning process and how they respond to the explanations and discussions. Activities given in the book help to make children participate in learning process more actively. By instructing and guiding the students to attempt the exercises like 'Try these', 'Think and Discuss' in the class room itself they must be encouraged to understand the concepts by firsthand experience, to discuss in groups and to express their opinions to convince the others. Then they must be evaluated how far they understood the concepts.

2. Class work, Homework, Portfolio's, Assignments :

By correcting class work and home work note books, portfolios and assignments teacher must estimate the students that how they are able to solve problems, gather information, record the data in proper formats, apply the mathematical reasoning. Teachers must verify class work note books to find whether the students are able to solve 'Do these' exercise by their own, home work note books to find how students solve the exercise problems, projects/assignments to find how the students gather information, analyse and record.

3. Slip Test :

Slip test may be conducted then and there the teacher wants to. Prior intimation need not be given to students. This must be conducted in the instructional period only. It may be conducted to test one or two concepts, but with entirely new problems. This is meant for evaluating the competencies; Problem solving and Reasoning and proof.

Whatever may be the evaluation, ultimate aim is to achieve the academic standards among the children. Accordingly activities, debates and discussions must be conducted and children must be assessed.

Principles to be noted while conducting Formative and Summative Evaluations:

- ◆ Previously students are evaluated only on the basis of written tests, that to with knowledge based questions. But in present students are to be evaluated in many ways of learning.
 - 1) Participation and Reflections of the students during teaching learning process.
 - 2) Class work, Homework, Portfolio's, Assignments etc.,
 - 3) Slip Test
 - 4) Children Projects.
 - 5) Written test based on Academic standards.

First four of the above are tools for Formative Evaluation, where as 4th and 5th are the tools for Summative Evaluation.

- ◆ CCE refers to a system of school based assessment that covers all aspects of students growth and development. Evaluation of student's growth and development is continuous process rather than an event, built into the total teaching learning process and spread over the entire span of academic session. For the entire process the teacher is more important. Teacher has to plan execute the teaching learning process according to academic standards and simultaneously evaluate the students with teacher made question papers.
- ◆ Generally question papers constitute of questions mainly to test the knowledge from the text book. Now academic standards are fixed to every subject and class and new teaching learning procedures are suggested to achieve those academic standards. So the evaluation should also aim at achieving the academic standards.
- ◆ Questions in tests or projects must provoke the students to think, explore, compare and contrast between the other subjects and daily life and encourage them to express their own experiences and views.
- ◆ Problem solving: Under this academic standard questions of verbal and non verbal, data comprehension, graphs, analysis and synthesis, constructions, etc. with complexity shall be asked. The complexity of a problem is depends on making connections among the various concepts of subjects and real life situations, number of steps, number of operations, context unravelling and nature of procedures.

- ◆ Reasoning and Proof: Under this academic standard questions must aim at reasoning between various steps, understanding and making mathematical generalizations and conjectures, understanding and justifying procedures, examining logical arguments, understanding the notion of proof, use inductive and deductive logic.
- ◆ Communication: Under this academic standard questions must evaluate skills of writing, reading and expressing mathematical notations (verbal and symbolic forms), Creating mathematical expressions, Explaining mathematical ideas in her own words like- a square is closed figure having four equal sides and all equal angles, Explaining mathematical logic.
- ◆ Connections: Under this academic standard questions must comprise Connecting concepts within a mathematical domain, Connecting mathematics to different subjects and with daily life, connecting concepts of different mathematical domains like data handling and arithmetic or arithmetic and space, connecting concepts to multiple procedures.
- ◆ Visualization and Representation: Under this academic standard questions must test interpretation and reading data in a table, number line, pictograph, bar graph, 2-D figures, 3-D figures, pictures, Making tables, number line, pictograph, bar graph, pictures and Making use of mathematical symbols and figures.
- ◆ While conducting Formative or Summative evaluations for different classes the following academic standard wise weightages are to be followed scrupulously.

Class	Description	Formative Evaluation					P	Summative Evaluation										
		Participation & Reflections	Note books	Projects	Slip tests	Total		Problem solving		Reasoning and Proof		Communication		Connections		Visualization Representation		Total
1-2	Weightage	20%	20%	20%	40%	100%		Oral	Written	Oral	Written	Oral	Written	Oral	Written	Oral	Written	100%
	Marks	10	10	10	20	50M		10%	40%	10%	-	-	10%	10%	-	10%	10%	50M
3-5	Weightage	20%	20%	20%	40%	100%		50%		20%		10%		10%		10%		100%
	Marks	10	10	10	20	50M		25		10		5		5		5		50M
6-9	Weightage	20%	20%	20%	40%	100%		40%		20%		15%		10%		15%		100%
	Marks	10	10	10	20	50M		40		20		15		10		15		100M

Principle to be followed while writing a question paper:

- ◆ Teacher must prepare question paper by him / herself for Summative Evaluation and for slip test in Formative Evaluation.
- ◆ Slip test in Formative Evaluation may be conducted once or in set of small test, totally for 20 marks.
- ◆ Question paper of Summative Evaluation should be prepared with different kinds of questions as per the following weightage table.

S. No	Type of Question	1 to 5 classes			6 to 9 classes		
		No of Questions	Marks for each	Total	No of Questions	Marks for each	Total
1	Essay	4	5	20	4	10	40
2	Short type	8	2½	20	8	5	40
3	Very Short type	5	1	5	10	1	10
4	Objective	5	1	5	20	½	10

Note: Objective type includes fill in blanks, multiple choice, true or false (for 1-5) and matching questions.

Academic standard wise weightage table for Summative Evaluation

Class	Description	Problem solving		Reasoning and Proof		Communication		Connections		Visualization Representation		Total
		Oral	Written	Oral	Written	Oral	Written	Oral	Written	Oral	Written	
1, 2	Weightage	10%	40%	10%	-	-	10%	10%	-	10%	10%	100%
	Marks	5	20	5	-	-	5	5	-	5	5	50
3, 4, 5	Weightage		50%		20%		10%		10%		10%	100%
	Marks		25		10		5		5		5	50
6 to 9	Weightage		40%		20%		15%		10%		15%	100%
	Marks		40		20		15		10		15	100

- ◆ Simultaneously the teacher must follow the academic standard wise weightages as follows. "

In summative evaluation question paper weightage for problem solving must be followed and weightages for the other academic standards may be altered according to the content to be tested.
- ◆ **Essay questions** must provoke the students to think, explore, compare and contrast between the other subjects and daily life and encourage them to express their own experiences and views. These questions must require descriptive answers. Theorems, constructions, solving equations and graphs may be included.
- ◆ **Short questions** must seek answers about one concept, figure, problem with 3 or 4 steps, little explanations only.
- ◆ **Very short answer** questions must be of problems with 1 or 2 steps, oral, definitions, formulae, symbols interpretation from tables or graphs, etc.
- ◆ Objective type questions must very very short and can be calculated in no time. These question must not be with multiple answers.
- ◆ All questions must be new (not from the text book) to the possible extent.
- ◆ In every academic standard essay type question must be given internal choice (one out of two should be answered).
- ◆ In any chapter if large questions are not available, two questions (a and b) may be given in each essay question.

How to consider the mistakes written by the students?

Continuous Comprehensive Evaluation is not a simple written test, but it refers to a system of school based assessment that covers all aspects of student's growth and development such as physical and mental development, social behavior, attitudes and interests. It includes both scholastic and co-scholastic areas and should be comprehensive in nature. This is in line with the goals of education. CCE is carried out simultaneously with the teaching learning process, i.e., in class room, laboratory, library, play ground, projects and other activities

During the teaching learning process when a student does mistakes in giving oral answers or representing in writing we don't point out much, instead we use to teach the same concept again in alternative mode and ask the student to repeat the study. In correcting note books or evaluating examination answer scripts it is not enough to correct the mistake(what we think), but we should enquire what is his / her idea in answering so. By this we can understand his draw backs such as i. Unable to understand language, ii. Don't know the concepts, iii. Misunderstanding the instructions and iv. Unable to analyse the problem. Then the student may be given proper guidance to learn.

Due to these reasons teachers must be cautious in using the terms, symbols, words and phases during teaching and writing the question paper. Instructions and questions must be in simple sentences.

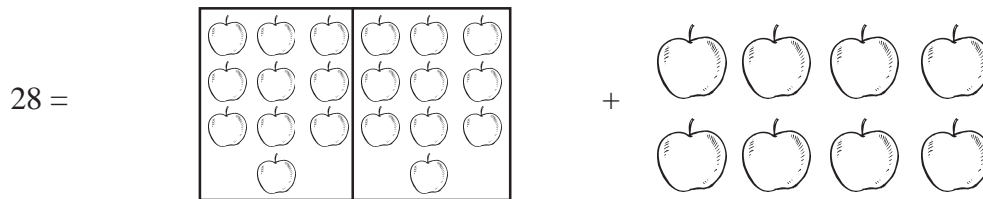
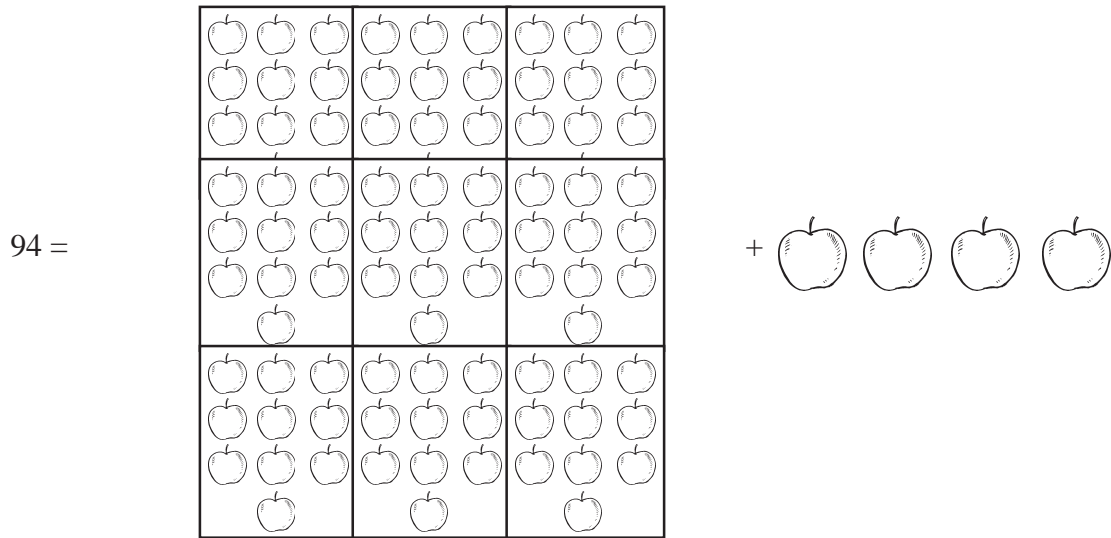
Students are unable to use their understanding in written tests. Why?

Generally students are able to use the mathematical concepts, what they comprehend in class room, in their daily life effectively. But they are unable to use the same competencies in writing a test. This is because the difficulty with correlation of mathematics with symbols, language writing procedures. In order to overcome this, pin pointed explanation, enough exercises and drilling must be given to them. Introduction mathematical terms and writing procedures must be explained with due care.

Consider an example. A student answered an addition problem as given aside.

$$\begin{array}{r}
 94 \\
 28 \\
 \hline
 1112 \\
 \hline
 \end{array}$$

Instead of giving the correct answer 122, the teacher must analyse the students understanding. The student is unable to identify the place values. If it is identified and proper reinforcement with concrete examples is given then the student does not do the same mistake again.



94 Apples	=	90 Apples	+	4 Apples
28 Apples	=	20 Apples	+	8 Apples
	=	110 Apples	+	12 Apples
<hr style="border: 0.5px solid black;"/>				
	=	110 Apples	+	10 Apples + 2 Apples
	=	120 Apples	+	2 Apples
Total Apples	=	122 Apples		

If it is explained as "Here sum of units places $4 + 8 = 12$ has one 10 and 2. This 10 has to be added to tens place", then students will definitely understand the concept.

Chapter - 8

Self Evaluation Sheet

Teacher's name : _____ Qualification : _____
 Designation : _____ School : _____
 Mandal : _____ District : _____

From your experience in the training program express your opinion for following questions.

	Extensive	Better	Satisfactory
1) Training has been given according to new text books.	()	()	()
2) Key principles of syllabus reflected in textual lessons.	()	()	()
3) Arrangement of chapters is scientific and gave equal importance to all the chapters.	()	()	()
4) Language and print in text books are appropriate to the students.	()	()	()
5) Understood the method of analyzing each chapter.	()	()	()
6) Different items incorporated in chapters are suitable to evaluate all academic standards.	()	()	()
7) Understood the method of teaching verbal problems.	()	()	()
8) Learnt what is probability and its scope.	()	()	()
9) Identified the need for Proofs in Mathematics.	()	()	()
10) Discussions were conducted about the approach papers on different areas of mathematics.	()	()	()
11) Understood the importance of irrational numbers and method of marking them on number line.	()	()	()
12) Learnt about the variable, equation and linear equation in algebra.	()	()	()
13) Understood the axiomatic approach to comprehend the geometry.	()	()	()
14) Trained to construct geometrical constructions properly.	()	()	()
15) Enhanced the understanding of different statistical methods and graphical representations.	()	()	()

- 16) Carried out the activities to understand areas and volumes. () () ()
- 17) Oriented to get prepared for teaching mathematics in new methods. () () ()
- 18) Knew the need for creating additional questions. () () ()
- 19) Discussed to evaluate each and every student as per CCE. () () ()
- 20) Training helped me to prepare question papers on the basis of academic standards. () () ()

- 1) Which session impressed you more? : _____
Why it impressed you? : _____
- 2) Which session did not satisfy your needs? : _____
Why do you think so? : _____

Date

Signature of the participant